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Deep Dive Safeguard: Advanced Searching for MCMS

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Abstract:

Maritime Cruises Mini-Submarines (MCMS) plans to offer a unique underwater experience in the Ionian Sea, contingent on regulatory approval and establishing safety procedures. We developed a Location Predictive Model using Runge-Kutta methods to predict submersible positions considering various uncertainties. Monte Carlo simulation handled random components. Recommended Search and Rescue (SAR) equipment, evaluated by Analytic Hierarchy Process, includes sonar systems and autonomous underwater vehicles, with the Remote Operated Vehicle being the most suitable. The Fast-Search Model optimizes SAR equipment deployment, reducing search time. The Destination Extension Model is adaptable to other tourist destinations. In conclusion, this article highlights critical location prediction and innovative SAR models supporting MCMS's application and approval, promising Greece leadership in maritime tourism with high safety standards.

Keywords: Differential Equation; the Runge-Kutta; the Analytic Hierarchy Process

1. Introduction

The Ionian Sea is known for its deep blue waters and is a significant body of water that separates the Italian Peninsula from the Balkan peninsula, stretching south from the Adriatic Sea. This sea is not only a crucial route for maritime trade but also a popular destination for tourism, offering spectacular coastal landscapes, rich historical sites, and diverse marine life.

Modern mini-submarines, are marvels of engineering designed to safely carry humans into the depths of the ocean. These vehicles are built to endure the extreme pressures found thousands of meters below the surface, enabling exploration of the vast majority of the world's ocean floors. They are equipped with sophisticated propulsion systems for precise navigation around complex underwater landscapes. Communication with the surface is achieved through acoustic signals, as radio waves cannot penetrate deep water, ensuring a constant link with the host ship or base of operations[1]. To ensure the safety and efficiency of these submersibles, they also feature with a range of emergency safety features. These include ballast control systems for emergency surfacing, redundancy in life support systems, and emergency locators to facilitate rescue operations if necessary.

In this paper, a model is established to predict the position of submersibles over time, and the associated uncertainties are identified. Determine what information a submersible can periodically transmit to the main ship to mitigate these uncertainties before an accident occurs, and specify the equipment needed for transmission. Then, based on the results of the model, the equipment requirements are analyzed, the location model is created, the probability of the positioning of the submersible is calculated, and the model is extended to other tourist destinations.

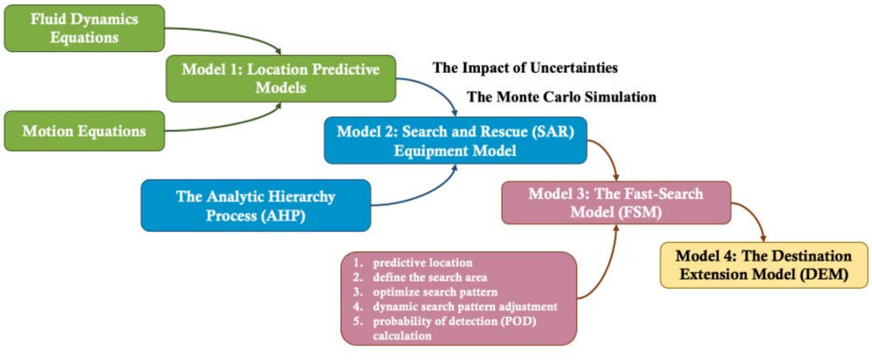

FIG. 1: Model Overview

2. Related Work

2.1 Assumptions

The submersible can control its thrust in three dimensions and can alter its buoyancy to ascend or descend. The search asset has a finite speed and can change direction instantly for simplification and environmental conditions and detection feedback are received in real-time.

2.2 Notations

Table 1: Data description

Symbol	Definition	Units
T	Constant thrust force	N
D	Drag force	N
B	Buoyancy force	N
G	Gravity	N
C	The current force	N
ρ	Density of seawater	kg / m^3
g	Acceleration due to gravity	m / s^2
A	Cross-sectional area of the submersible	m^2
C_d	Drag coefficient	--
V	Volume of the submersible	m^3
m	Mass of the submersible	kg
$\vec{T}$	The thrust vector generated by the propulsion system	N
$\vec{D}$	The drag force vector	N
$\vec{G}$	The gravitational force vector acting on the submersible	N
$\vec{B}$	The buoyancy force vector	N
$\vec{C}$	The current force vector	N
?V	The change in volume due to ballast adjustments.	m^3
$\vec{p}$	The position of submersible	m
$\overrightarrow{?_c}$	Random vector representing the uncertainty in current prediction	--
p	Pressure of sea water	N
$\overrightarrow{?_n}$	The navigation error	--
P_{search}	The search pattern	--
λ	Weighting factor balancing search time and probability of detection POD	--
A_{search}	Search area	m^2
$D(t)$	Real-time data received at time t, including any sightings, sensor readings, or environmental updates	--
α	The detection rate	--
s_i	The submersibles	--

3. Location Predictive Models (LPM)

3.1 Establishment of LPM

The descent of Maritime Cruises Mini-Submarines (MCMS) in seawater is a physical process influenced by the interplay of multiple forces. Therefore, we construct a mathematical model that predicts the position of a submersible over time based on its equations of motion and fluid dynamics. To further analyze this problem, we consider several factors to establish a preliminary predictive model in a 2D environment, analyzing the submersible moving horizontally (x-axis) and vertically (y-axis) in water[2].

Drawing on principles of physics, the basic forces acting on the submersible include thrust (T), drag (D), buoyancy (B), and gravity (G). The equations of motion for the submersible in the horizontal and vertical directions can be given as:

For horizontal motion (x-axis):

$$m\frac{d^2x}{dt^2}=T-D$$

For vertical motion (y-axis):

$$m\frac{d^2y}{dt^2}=B-G-D$$

where: m is the mass of the submersible; T is the thrust generated by the propulsion system.

Additionally, D is the drag force, which can be calculated by $D=\frac{1}{2}\rho v^2 C_d A$, where: ρ is the water density; v is the velocity of the submersible; C_d is the drag coefficient; A is the cross-sectional area of the submersible.

B is the buoyancy force, equal to the weight of the water displaced by the submersible, $B=\rho gV$, where: g is the acceleration due to gravity; V is the volume of the submersible.

G is the gravitational force acting on the submersible, $G=mg$.

Furthermore, we will consider differentiating v into two dimensions, where v_x, v_y represent the velocities of the submersible in the horizontal and vertical directions, respectively. For this model, with reference to the conventional parameters of a deep submarine, the initial parameters are selected as follows: $\rho=1025kg/m^3, A=1.0m^2, g=9.81m/s^2$,

$C_d=0.5, V=5.0m^3, m=2000kg, T=5000N$.

Based on these parameters, we calculate the following model results (as shown in FIG. 2). Under normal circumstances, the submersible will descend rapidly after entering the surface until it reaches the required depth of water.

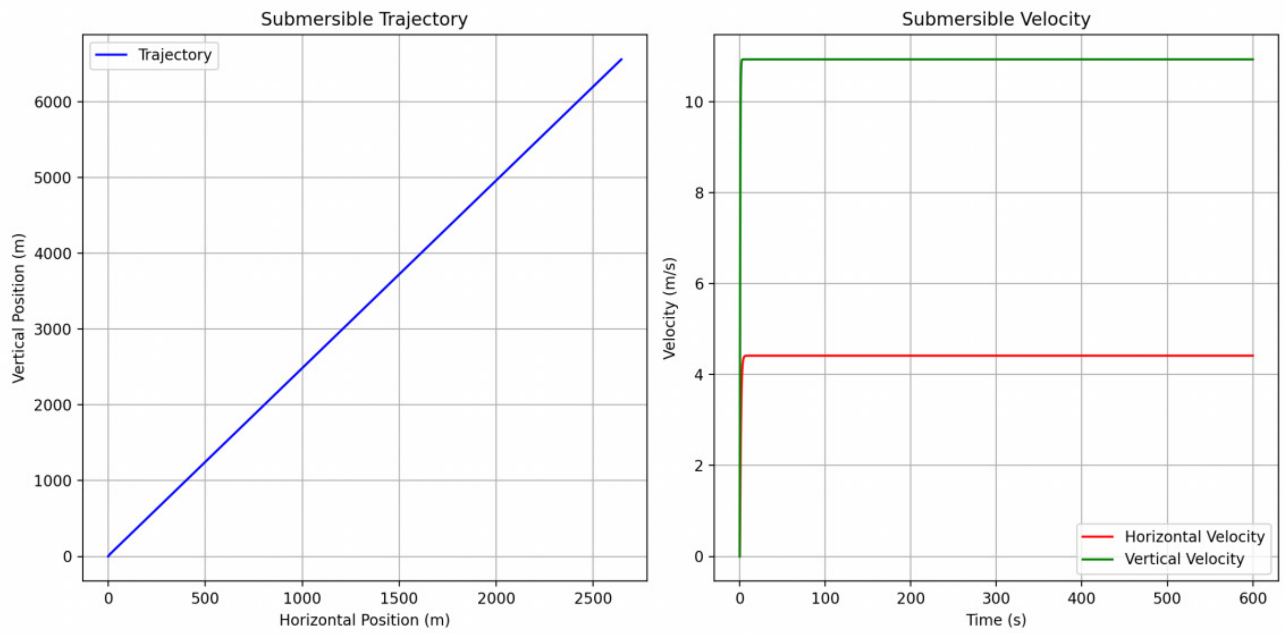

FIG. 2: Basic Model of 2D Prediction

For a more accurate construction of a submersible's predictive model that accounts for three-dimensional movement and variable conditions, we'll include the effects of ocean currents, varying thrust, and the possibility of changing buoyancy. We'll assume the submersible can control its thrust in three dimensions and can alter its buoyancy to ascend or descend.

When we consider the submersible is affected by an ocean current, which has both a known magnitude and direction at different depths and positions, the equations of motion now become:

$$m\frac{d\vec{v}}{dt} = \vec{T} - \vec{D} - \vec{G} + \vec{B} + \vec{C}$$

where: $\vec{v}$ is the velocity vector of the submersible. $\vec{T}$ is the thrust vector generated by the propulsion system. $\vec{D}$ is the drag force vector, now dependent on the relative velocity of the submersible to the water around it, which includes the effect of the current. $\vec{G}$ is the gravitational force vector acting on the submersible. $\vec{B}$ is the buoyancy force vector, which can be adjusted by changing the submersible's volume (for example, by using ballast tanks). $\vec{C}$ is the current force vector, which depends on the current's velocity profile.

The drag force is still calculated with the same equation but now applied to each component of the relative velocity between the submersible and the surrounding water:

$$\vec{D} = \frac{1}{2}\rho(\vec{v} - \vec{v}_{current})^2 C_d A$$

The current force will be determined from oceanographic data and is a function of position and depth:

$$\vec{C} = f(\vec{p}, depth)$$

The thrust vector will be a function of the submersible's control inputs:

$$\vec{T} = f_{thrust}(control\_inputs)$$

The adjusted buoyancy can be modeled as:

$$\vec{B} = \rho g ?V$$

where $?V$ is the change in volume due to ballast adjustments.

To solve this system of equations, we use a method like the Runge-Kutta (the 4th order, RK4). Furthermore, we update the plot building on the initial conditions by incorporating more realistic conditions, depicting the submersible's position over time with variable thrust and buoyancy. This graph presents the submersible's horizontal movement (x-position) and depth (z-position) as functions of time. The sinusoidal variations in thrust and buoyancy result in fluctuations in the trajectory. The results are shown in FIG. 3.

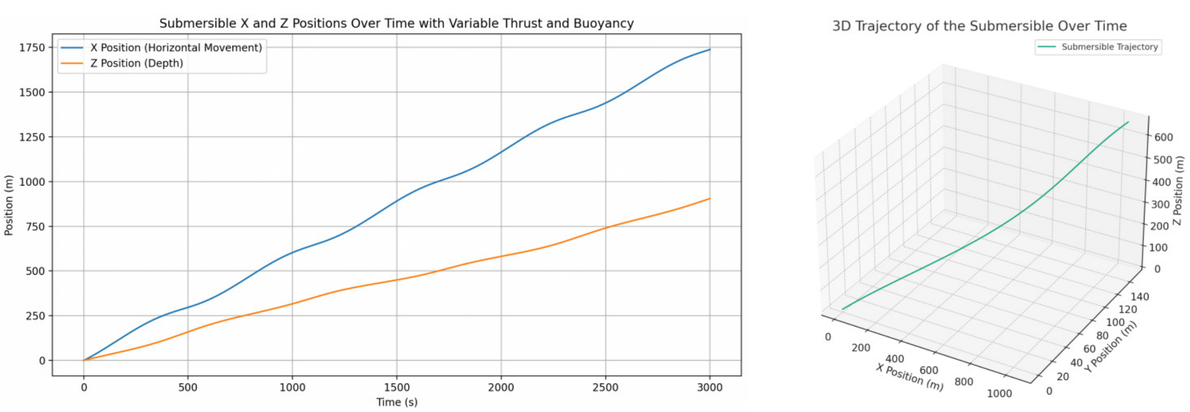

FIG. 3: Positions Over Time and 3D Trajectory

3.2 The Impact of Uncertainties

Before an incident occurs, a submersible can regularly transmit a variety of critical information to the host ship to reduce operational uncertainties.Incorporating the uncertainties into the model requires modifying the equations to integrate stochastic elements and environmental data. Therefore, here are the extended mathematical formulations for the model:

The force due to currents $\vec{C}$ can be modeled based on environmental data, which varies with position and time. Introduce a stochastic term to account for the unpredictable nature of currents.

$$\vec{C}(t, \vec{p}, depth) = \vec{C}_{data}(t, \vec{p}, depth) + \overrightarrow{?_c}$$

Where $\vec{C}_{data}$ is the known current vector from data, and $\overrightarrow{?_c}$ is a random vector representing the uncertainty in current prediction.

Additionally, temperature t and pressure p affect the water density ρ, which in turn affects buoyancy $\vec{B}$. A model for this relationship might be:

$$\rho(t, p) = \rho_0[1 + \alpha(t - t_0) + \beta(p - p_0)]$$

Where α and β are coefficients for the temperature and pressure effects respectively, and ρ_0, t_0, p_0 are reference values.

Propulsion system failures can be modeled as a binary random variable that affects the thrust $\vec{T}$, while navigation errors add noise to the position estimate.

$$\vec{T}(t, controlinputs) = \begin{cases} 0, withprobabilityp_{fail} \\ \vec{T}_{input}(controlinputs), otherwise \end{cases}$$

In this case, p_{fail} represents the probability that the propulsion system will fail at any given time, resulting in no thrust output. When the system is functioning properly, the thrust $\vec{T}_{input}$ is determined by the submersible's control inputs.

For the estimated position $\vec{p}_{est}$ with navigation errors:

$$\vec{p}_{est} = \vec{p}_{true} + \overrightarrow{?_n}$$

Here, $\vec{p}_{true}$ is the true position of the submersible, and $\overrightarrow{?_n}$ represents the navigation error, which can be modeled as a random variable with a probability distribution (often a normal distribution) to simulate the inaccuracy in the submersible's navigation system.

Terrain can be represented by a function $T(\vec{p})$ that impacts navigation, requiring adjustments to avoid collisions.

$$\vec{F}_{terrain} = \nabla T(\vec{p})$$

Interactions with marine life introduce random perturbations $\vec{F}_b$ in the path of submersible.

$$\vec{F}_b = randomforceduetobiologicalfactors$$

Putting it all together, the equations of motion incorporating these uncertainties can be written as:

$$m\frac{d\vec{v}}{dt} = \vec{T} - \vec{D}(\vec{v} - \vec{C}) - \vec{G} + \vec{B}(\rho(t, p)) + \vec{F}_{terrain}(\vec{p}) + \vec{F}_b(t)$$

The equations introduce environmental factors, technical issues, seabed terrain challenges, and biological interferences. To solve these numerically, we use the Monte Carlo simulation to handle the random components.

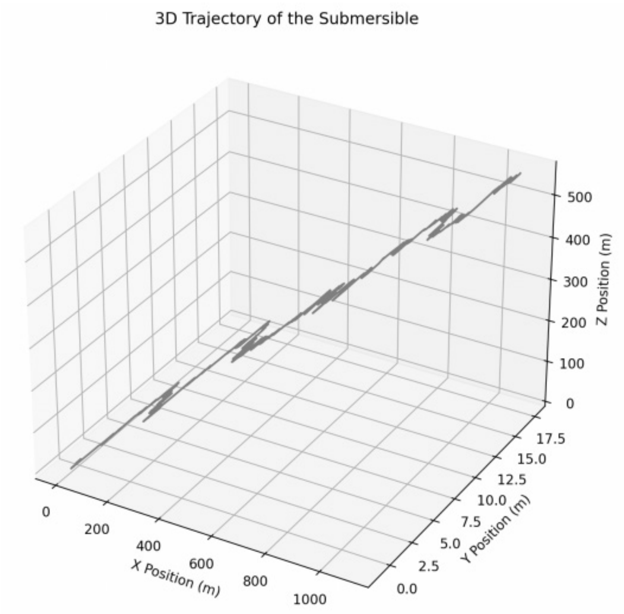

FIG. 4: The 3D Trajectory Based on the Impacts of Uncertainties

4. The Search and Rescue (SAR) Equipment Model

4.1 Search Equipment

There is some additional search equipment, along with considerations of their cost, availability, maintenance, readiness, and usage, shown in Table 2.

Table 2 The Additional Search Equipment

Equipment	Considerations
Remote Operated Vehicle (ROV)	1. Can be remotely controlled to conduct underwater searches with cameras and sonar. 2. High initial investment but can be offset by versatility and reduced risk during SAR operations.
Autonomous Underwater Vehicle (AUV)	1. Can cover large areas autonomously, equipped with side-scan sonar and other sensors. 2. AUVs are expensive but provide extensive search capabilities.
Side-scan Sonar	1. Provides detailed images of the seafloor and can identify anomalies indicative of a submersible. 2. Operating costs are moderate, and the equipment is relatively easy to maintain.
Towed Pinger Locator (TPL)	1. Used to detect acoustic signals from the submersible's emergency beacon. 2. Low operational cost and can be deployed quickly, essential for initial search phases.
Dive Teams with Mixed Gas Diving Equipment	1. Human divers can perform close inspections and participate in potential rescue operations. 2. Diving operations are expensive and pose risks to the divers; thus, they are typically used after locating the submersible.

Underwater Communication Systems	1. Enables communication between surface team, ROV/AUV, and potentially trapped submersible if within range. 2. Requires investment in multiple units for redundancy and training for operators.
Emergency Life Support Pods	1. Can be sent down to supply trapped personnel with air and essentials while a rescue is being organized. 2. Moderate cost, reusable, and require regular maintenance checks.
Acoustic Beacons and Transponders	1. Aid in relocation of the submersible if it has drifted or if initial search efforts were unsuccessful. 2. Relatively low cost and can be a standard part of the safety equipment.

4.2 Selection Considerations

To evaluate the search and rescue equipment for a submersible operation, we conduct **the Analytic Hierarchy Process (AHP)** and first establish a hierarchy of decision criteria, compare the equipment pairwise, and then rank the equipment options based on these criteria.

Table 3 The Decision Criteria

Criteria	Sub-Criteria
Cost	Initial cost, operational cost, and cost per use.
Availability	Lead time for purchase, frequency of use, and rental options.
Maintenance	Regular maintenance needs, complexity, and cost of maintenance.
Readiness	Time to deploy, ease of deployment, and operator training required.
Usage	Versatility, effectiveness in SAR operations, and historical success rates.

The structure of the analytic hierarchy process model is shown as follows:

In the process of evaluating the optimal search and rescue (SAR) equipment for submersible operations, we have determined the relative importance of various criteria based on real-world considerations. The criteria weights, which reflect the priorities for selecting SAR equipment, are as follows:

· Cost: 0.25
· Availability: 0.20
· Maintenance: 0.15
· Readiness: 0.25
· Usage: 0.15
· At the same time, we evaluated the parameter values of each device, as shown in Table 4.

Table 4 The Scores of Different Equipment

Equipment	ROV	AUV	Side-scan Sonar	TPL	Dive Teams
Cost	0.15	0.10	0.25	0.30	0.20
Availability	0.25	0.20	0.15	0.20	0.20
Maintenance	0.20	0.25	0.30	0.15	0.10
Readiness	0.30	0.20	0.10	0.20	0.20
Usage	0.20	0.30	0.15	0.20	0.15
Overall Scores	0.2225	0.1975	0.185	0.2175	0.1775

This result suggests that, given the weights for cost, availability, maintenance, readiness, and usage, the ROV is the most suitable SAR equipment according to the criteria set in this scenario.

5. The Fast-Search Model (FSM)

5.1 Establishment of FSM

To establish a model that utilizes submersible location data to optimize the deployment of SAR equipment and calculates the probability of detection over time, we propose the following analytical framework. This model integrates predictive analysis based on the last known location and trajectory of the submersible, environmental conditions, and the operational parameters of the SAR equipment.

Step 1: the predictive location analysis

It aims to estimate the current and future positions of a submersible based on its last known location, velocity, and environmental factors such as ocean currents. We assume that the submersible moves according to its last known velocity V_{sub} and is influenced by ocean currents C. The ocean currents C are assumed to be a vector field that varies with location and time.

We calculate the probable current location.

$$L_{current} = L_{last} + V_{sub} * t + \int_0^t C(t)\,dt$$

L_{last} is the last known location vector of the submersible (latitude, longitude, and depth). V_{sub} is the velocity vector of the submersible at the last known location. t is the time elapsed since the last known location. $C(t)$ represents the ocean current vector as a function of time t, affecting the submersible's trajectory. The integral $\int_0^t C(t)\,dt$ calculates the cumulative effect of the ocean currents over time, from the moment of the last known position to the current time.

Step 2: define the search area

Defining the search area for a lost submersible involves estimating the extent over which the submersible could have moved since its last known position, taking into account its own mobility and the influence of environmental factors like ocean currents. The total maximum possible displacement D_{total} is the sum of D_{sub} and $D_{current}$. This defines the radius of the search area centered at the last known location or the projected current location $L_{current}$. The model to define the search area A_{search} can be formulated as follows:

$$D_{total} = D_{sub} + D_{current} = |V_{sub}| * t + \int_0^t C(t)\,dt$$

$$A_{search} = \pi * (D_{total})^2$$

This model provides a structured approach to estimating the search area for a lost submersible, crucial for effective planning and execution of search and rescue operations.

Step 3: optimize search pattern

We further use an optimization algorithm to minimize the objective function $f(P_{search})$, which is defined as:

$$f(P_{search}) = Timetocompletesearch - \lambda * POD$$

Where: P_{search} is the search pattern, λ is a weighting factor balancing search time and probability of detection POD, Time to complete search is the total time taken by the SAR equipment to follow P_{search} and search the area A_{search}, POD is the probability of detection.

Step 4: dynamic search pattern adjustment model

To construct a sophisticated model for dynamic search pattern adjustment, we need to consider real-time data acquisition and feedback mechanisms that allow the search pattern to adapt to new information dynamically. Firstly, we assume the search asset has a finite speed and can change direction instantly for simplification and environmental conditions and detection feedback are received in real-time.

We set $P_{search}(t)$ as the search pattern at time t, which is a sequence of waypoints or vector field that the SAR asset will follow. $D(t)$ means real-time data received at time t, including any sightings, sensor readings, or environmental updates. V_{SAR} means the velocity of the SAR asset. $C_{env}(t)$ means the updated environmental conditions, such as current changes.

The formulation is,

$$P_{search}(t + ?t) = Adjust(P_{search}(t), D(t), C_{env}(t))$$

$?t$ is the time increment for the update.

The Adjust function recalculates the search pattern based on new data and environmental conditions.

$$W_{new} = RecalculateWaypoints(P_{search}(t), D(t), V_{SAR}, C_{env}(t))$$

This function generates a new set of waypoints W_{new} that the SAR asset will follow.

$$f(P_{search}, D, V_{SAR}, C_{env}) = min(Expectedtimetodetection)$$

The objective function minimizes the expected time to detection based on the current search pattern, real-time data, SAR asset velocity, and environmental conditions.

Step 5: probability of detection (POD) calculation

The POD can be calculated at each step of the search based on the area covered and the effectiveness of the SAR equipment:

$$POD(t) = 1 - e^{-\alpha * A_{covered}(t)}$$

α represents the detection rate per unit area covered by the SAR equipment. $A_{covered}(t)$ is the cumulative area covered by the search operation up to time t.

This Fast-Search Model (FSM) provides a structured approach to optimizing SAR operations for submersibles by integrating predictive analytics, environmental data, and operational capabilities of SAR equipment. The dynamic nature of the model allows for real-time adjustments to search patterns, enhancing the efficiency and effectiveness of the search efforts.

5.2 Interpretation of Results

The plot below illustrates the initial and adjusted search patterns based on a hypothetical dynamic search pattern adjustment scenario. The blue spiral represents the original search pattern centered around the initial search center (red marker). The green dashed spiral represents the adjusted search pattern, shifted to a new center (orange marker) due to simulated new data indicating a possible sighting or other relevant information.

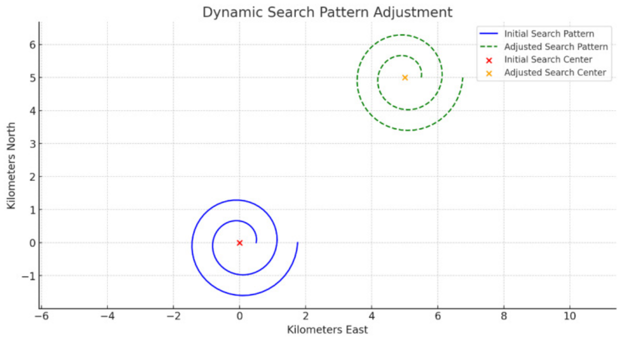

FIG.4: Search Patterns

Furthermore, POD calculation model is initialized with a set of given parameters to simulate the search and rescue operation over a defined p5eriod and area. The detection rate per unit area, denoted by \[\alpha \], is set to 0.1, which represents the efficiency of the search equipment in detecting the target per square kilometer of area searched. The total area designated for the search operation is 100 square kilometers. For the purpose of modeling and visualization, the search duration is discretized into 100-time steps, which allows us to plot the probability of detection against time and observe how it evolves as the search progresses.

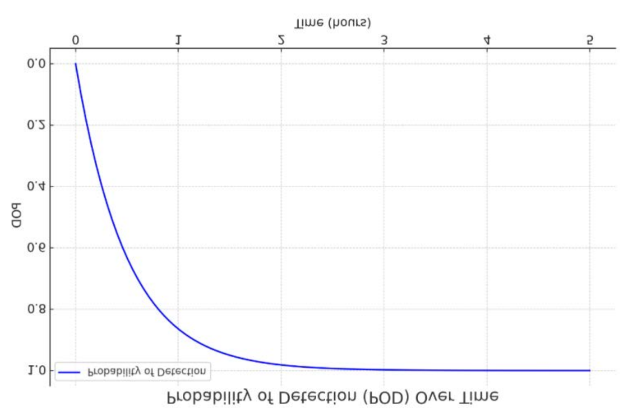

Fig. 5: The Simulation of POD Model

In this simulation, we've used a linear coverage model where the search area is covered uniformly over time. As time progresses, the cumulative area covered increases, thereby increasing the POD, as reflected in the FIG. 5.

6. The Destination Extension Model (DEM)

6.1 Establishment of DEM Model

To adapt the FSM model for broader use in areas like the Caribbean Sea, we would focus on the revision in the blow aspects: Environmental Adaptation、Legal and Regulatory Compliance、 Cultural and Economic Factors、Infrastructure and Support

Furthermore, when dealing with multiple submersibles in the same locations, a comprehensive model must be established to ensure efficient operation and safety. The specific formulas are as follows:

(1) Tracking and Coordination:

Let $S=\{s_1,s_2,s_3,\cdots,s_n\}$ be the set of submersibles. And each submersible s_i reports its position $P_i(t)$ at time t. The coordination model ensures no overlap in search regions $R_i(t)$ for any two submersibles ??s_i and s_j:

$R_i(t)\cap R_j(t)=\varnothing, \forall i \neq j$

(2) Communication Network

Define a graph $G=(V,E)$ where vertices V represent submersibles and edges E represent communication links. The communication model ensures message delivery M from submersible s_i to the host ship H within a time ?t:

$M_{s_i \to H}(t+?t)$

(3) Collision Avoidance:

For any two submersibles s_i and s_j, maintain a minimum safe distance d_{safe}.

¡¬$P_i(t)-P_j(t)$¡¬$\geq d_{safe}, \forall i \neq j$

(4) Search Pattern Complexity:

Divide the search area A into sectors $A=\{A_1,A_2,\cdots,A_n\}$ and assign them to submersibles. The search pattern for submersible s_i over time t is $P_i(t)$, covering sector A_i.

(5) Probability of Detection Adjustments:

The cumulative POD for all submersibles at time t is:

$$POD(t)=1-\prod_{i=1}^{n}(1-POD_{s_i}(t))$$

Where $POD_{s_i}(t)$ is the POD for submersible s_i based on its covered area up to time t.

6.2 Stimulation Results

Furthermore, we establish initial conditions to model a search pattern for DEM operations in the Caribbean Sea. We set the strength of ocean currents to 2 km/h and apply a water clarity factor of 0.8 to adjust the detection rate, reflecting the clear waters typical of the region. The search area is conceptualized as a 50×50 km grid, representing a simplified version of the Caribbean Sea environment.

The model simulates a submersible's search operation by executing a random walk, starting from the center of this grid. Over a series of 200 steps, the submersible randomly moves through the grid, with each cell visit incrementing its value, which is indicative of the search intensity in that area. The detection rate is set at a base value of 0.1, which is then modified by the water clarity factor to calculate the POD for each cell. The simulation results in two visualizations: one showing the search grid with the number of visits to each cell, and the other depicting the POD across the search area (FIG.6).

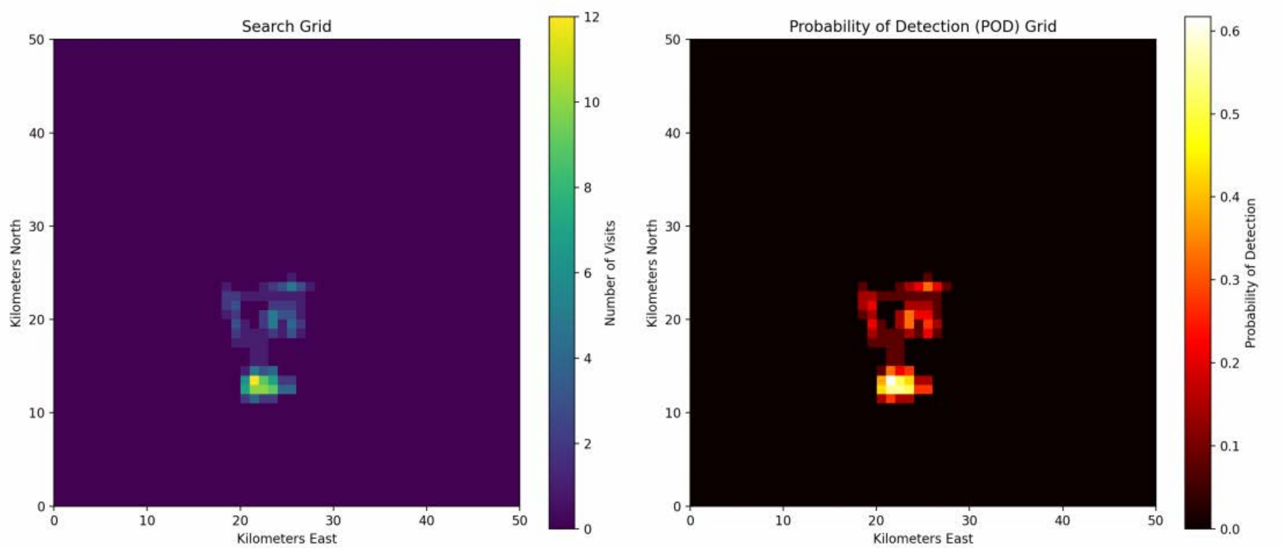

Fig. 6: The Simulation of DEM

7. Conclusion

7.1 Summary of Models

The suite of models designed for submersible search and rescue (SAR) operations offers a robust framework for predictive analysis, search optimization, and adaptability to environmental conditions. They excel in focusing search efforts, optimizing resource allocation, and providing real-time responsiveness to changing data. However, these models are data-dependent, requiring accurate and timely information, and can be computationally demanding. Their simplified assumptions may not fully encapsulate the complexities of marine environments, and they necessitate continuous refinement to ensure efficacy and reliability in diverse operational scenarios.

In summary, these models provide a comprehensive framework for enhancing SAR operations for submersibles. While they offer structured methodologies and can substantially improve search effectiveness, their performance is inherently tied to the quality and timeliness of data, computational resources, and the real-world complexity of marine environments. The models necessitate continual refinement and validation against real-world SAR scenarios to ensure their robustness and reliability.

7.2 Future Improvements

To advance submersible SAR operations, future improvements should focus on enhancing data quality through high-resolution, real-time environmental inputs and leveraging machine learning for predictive model optimization. Developing realistic simulations and incorporating cutting-edge sensor and communication technologies will bolster detection and coordination capabilities. Adapting models to comply with evolving legal frameworks and understanding cultural impacts are crucial for global applicability. Emphasizing collaborative efforts with SAR organizations and fostering a culture of continuous learning and adaptation will ensure models remain current and increasingly effective. These strategies aim to refine SAR operations, making them more precise, efficient, and adaptable to the dynamic challenges of marine environments.

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