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Exploring High-Dimensional Spaces and Their Real-Life Applications

Yinpu Gong

Beijing Royal School, Beijing, 102209, China
zhongtaibwfs@163.com

Abstract:
Understanding high-dimensional spaces is crucial for various applications in mathematics and physics. The article intended to discover the relations between high-dimensional space and its applications in real life, based on the theories mentioned before. The research will be divided into two large parts and each part contains two objectives, which depend on the depth of knowledge. The first part includes a discussion of the known dimensional space. It will present simple explanations of these dimensional spaces. Next, the differences of adjacent dimensional space will be included. In addition, it will discuss the relationship in the known dimensional space. In the second part, the paragraph will try to transfer 4D or higher dimensions from 3D and existing data, which is the most significant objective. At the end of the research, it will find some applications related to the research in daily life, hoping more people will realize what high dimensional space is and associate it with daily life.

Keywords: mathematics, physics, position vector, dimensional space.

1. Introduction

The higher-dimensional technology is used to provide possible explanations of amazing phenomena, like the universe or brain. This technology also has certain applications in life, although the four dimensions have not been confirmed.

Humans live in three-dimensional space, and the presence of height, width, and depth allows organisms to inhabit that space comfortably. However, the sense of the passage of time in everyday life goes beyond the three-dimensional realm, suggesting the existence of a different dimension.

A lot of mathematicians and physical experts have explored the multiple dimensions. Cayley identified that the multiple variables behave similarly to multiple dimensions [1]. This work significantly contributed to the initial understanding of high-dimensional spaces. In 1846, Julius Plucker created a theory about using the extension, if a point is in three dimensions, then is has four homogeneous coordinates, as if the three-dimensional point were a projection onto 3D from a 4D space. This idea helps follow-up exploration of the concept of multi-dimensions. And it makes the research of high-dimensional space more ordinary [2].

Schlafi was the first person who have the full study of higher geometry dimensions. He described the angle point in multi-dimensions. He investigated high-di-

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mensional polytopes from which are derived from the modern “Schlafi notation” which is a symbol to illustrate tessellations and the number of polytopes per line or point. In 1854, Bernhard Riemann played a most significant role in a shift of mindset that eventually accepted the geometry of high-dimensional spaces by speech which uses a series of coordinates to describe any dimensional spaces [3]. In the 1920s, Theodore Kaluza added a new spatial dimensional with gravitational force and electromagnetic force. This is the first time of 5D construction. This trying filled people’s limited ideas about higher dimensions [4]. In 1932, an infinite dimension vector space called Hilbert space is created by John von Neumann. It used methods of linear algebra and calculus to be generalized from Euclidean vector (finite dimensions) spaces to spaces that may be infinite-dimensional [5]. In 1935, Einstein and Nathan Rose published a theory of the Einstein-Rosen bridge to solve two problems that are Schwarzschild singularity at radius r= 2M/c2, and applied the theory of general relativity to points like electrons. In 1995, Edward Witten’s observation led to the second superstring revolution until now. In this theory, 6 extra spatial dimensions are wrapped up into complex manifolds such as the Calabi-Yau manifold. There is something wrong with the three-plus-one dimensions of spacetime [6].

Those scientists and mathematicians made a dramatic contribution to the exploration of dimensional space. And the Einstein-Rosen bridge which is also called Wormhole is one of the strangest things in spacetime theory. It connects two points directly in the same spacetime whether how long distance or time. In summary, this paper hopes to analyze the connections of each adjacent known dimension and figure out correlations of these factors. Afterward, the article will predict the transformation of unconfirmed dimensions from low-dimension assumptions and further explore the application of dimensional space in real life.

2. Definition

2.1 Zero-Dimensional Space

The first step is having a clear understanding of the zero to three-dimensional spaces for transition to high-dimensional spaces. A zero-dimensional space is a topological space that has dimension zero about one of several inequivalent conceptions of distribution of a dimension to a given topological space which defines convergence, continuity, and communication formally in mathematics. More specifically, zero-dimensional space is a point that is a vague, idealized, and exact position or coordinate without taking any space, in the physical world. Point also exists in higher dimensional spaces or represents typical sets. In classical Euclidean geometry, it is a basic unit to define no body or part thing initially. In a plate, a point illustrates an ordered pair (x, y). X and y often represent the horizontal position and vertical position respectively. For 3D Euclidean geometry, the point represents the arrangement triple (x, y, z) additional number-z shows the depth. Further generalization, the point represents more terms in higher dimensions.

2.2 One-Dimensional Space

One-dimensional space is a mathematical space that can be identified by a single coordinate. For example, the line is a kind of one-dimensional space, every point in this line has a corresponding location with a single figure. Straight lines and smooth curves are both one-dimensional spaces whether the dimension of the topological space. In other words, a line that does not construct a coordinate system is a one-dimension. Meanwhile, the straight line is an infinitely long object without width and depth, which is only an infinite extension with two sides. Furthermore. a line has one direction, like left or right. Generally, points are the fundamental element of lines without surface and volume. Thus, the line is an idealization in our lives because there are no other cases that only have length. For example, hair looks like a line or a curve, however, hair is a three-dimensional product, because it has all the features of a 3D item: length, width, and radius.

2.3 Two-Dimensional Space

Two-dimensional space is a mathematical space with two dimensions, which means points have two degrees of freedom. Their position is described by two coordinates. In other words, they can move in two different directions. Normally, two-dimensional space is called the plane or the surface. “The most basic example is the flat Euclidean plane, an idealization of a flat surface in physical space such as a sheet of paper or a chalkboard. On the Euclidean plane, any two points can be joined by a unique straight line along which the distance can be measured. The space is flat because any two lines transited by a third line perpendicular to both are parallel, meaning they never intersect and stay at a uniform distance from each other.” When people want to get an exact location on the earth, it normally uses two units to show it: longitude and latitude, hence, the sphere is two dimensions.

In mathematics, the polar coordinate system is an alternative two-dimensional way to illustrate the location by describing angle and radius. The radius is the distance between the point and pole (often origin O), and the angle is measured anticlockwise from the initial line (often x-axis), polar coordinates are written as (r, θ). The angles in polar coordinates are represented by radians or degrees (2π or 360°). People usually use degrees in application regions, like direction, and radians are more ordinary in mathematics and physics. The polar coordinates system is the most suitable in almost all situations in which being considered is internal tried to direction and distance from the center point. It is usually simpler and more common to build a model.

It can convert polar coordinates to Cartesian coordinates by using right angle triangle trigonometry. As the figure 1 shown, $rcos\theta = x$; r sin θ = y; $r^2 = x^2 + y^2$; $\theta = arctan\left(\frac{y}{x}\right)$.

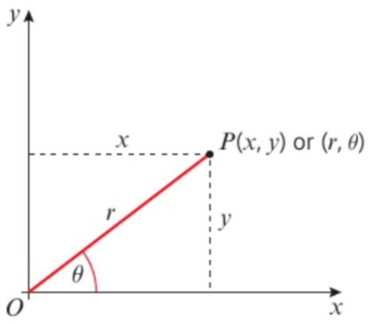

Fig. 1 polar coordinate (Photo/Picture credit: Original).

2.4 Three-Dimensional Space

At the end of the first step, three-dimensional space is the last confirmed dimension. Furthermore, To determine a location within space, an additional variable, altitude above the Earth, must be considered. Therefore, mathematics requires three variables to accurately define a position, categorizing space as three-dimensional. It can make sense that the construction of three-dimensional spaces in daily life. Three-dimensional space can accommodate lower dimensional spaces, and the length, width, and depth illustrate the distance relations compared to the original point O. In addition, people can regard a flattened cloth as a two-dimensional space. When humans make a weight-free fall, the cloth will be depressed by the gravitational force, which forms a three-dimensional space. In geometry, a three-dimensional space is a mathematical space that contains three coordinates to recognize the location of a point. Those coordinates of the figure can be understood as the Cartesian coordinates (figure 2) of location in a n-dimensional Euclidean space. It uses (x, y, z) to represent the accurate position in three-dimensional space. For example, the length, width, and depth of a cube is represented by x, y, z respectively. Those three coordinates are perpendicular to each other at original point (0, 0). In the system, the location of any points is given by ordered group of real number, and each figure illustrates the distance from position to original point, which is same as the point from the confirmed plane by other two axes. There are still other approaches to describe the three-dimensional spaces, like cylindrical coordinates (figure 3) and spherical coordinates (figure 4). A cylindrical coordinate system is a kind of 3D coordinate system which recognizes the position of point by the distance from choosing a fixed line, the direction from the axis relative to the selected fixed direction, and distance perpendicular to the axis from the selected certain plane. In the meanwhile, another distance depends on the side of plane facing the point by positive and negative polarity of number. In addition, a spherical coordinate system also is a 3D coordinate system. The position of certain point in system is identified by three figures (r, θ, φ). The distance of radial line r connecting the point and origin point.

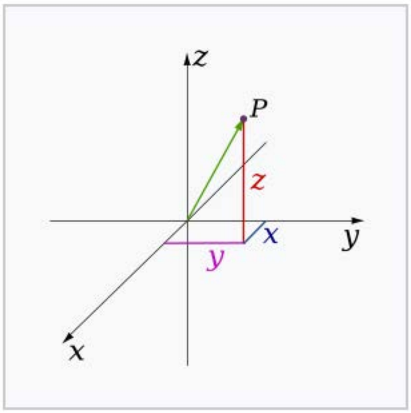

Fig. 2 Cartesian coordinate system (Photo/Picture credit: Original).

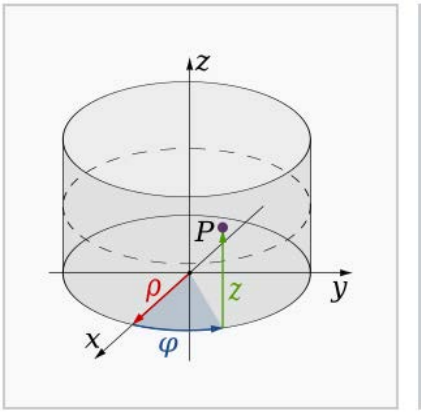

Fig. 3 Cylindrical coordinate system (Photo/Picture credit: Original).

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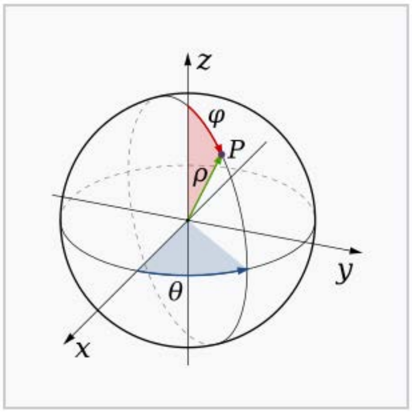

Fig. 4 Spherical coordinate system (Photo/ Picture credit: Original).

3 Features of Lower-Dimensional Spaces

To transfer to higher dimensional space, readers should have a clear awareness of the difference between confirmed dimensional spaces. The number of variables is the most obvious distinction of dimensional spaces, which means the higher dimensional spaces have more independent variables to represent the coordinate. One more variable existing in coordinate means the dimension of space increases to the next level. Generally, the dimension depends on the number of lines which is perpendicular to a common point. For example, two-dimensional space has 2 lines perpendicular to each other at one point. In this view, the points in zero-dimensional space have isolated each other, because a one-dimensional line is formed when it connects two points. Next, a point doubles the number of points, and connects each point, and a two-dimensional face is formed. Similarly, double the points again, and a three-dimensional space is formed.

At the beginning of the transformation, it starts at a point without size and dimensions, which is just an idea for the indicator of position in the coordinate system. If there is another point, that limits a size, it will create a one-dimensional space by connecting the two points. Adding a point that is not located same line as the first two can form a two-dimensional space, like a triangle. Also, mathematicians can get the same dimensions by a line, the difference is a rectangle formed. There is an interesting suggestion about achieving three-dimensional spaces. It accords to distort the middle part of two-dimensional space; this mind might benefit for imagination of higher dimensional space.

3.1 Exploration the High-Dimensional Space

If the mentioned theories or minds are still useful, four-dimensional space is formed by three-dimensional extension. A cube extent a space to create the four-dimensional space, however, it only can draw a picture of it on the plane or build a three-dimensional model to represent it. More significantly, people should distinguish the difference between four-dimensional space and four-dimensional spacetime. At first, they are not the same things, because dimensions have different meanings in mathematics and physics. In mathematics, any variables can be the dimensions, such as x, y, z the fourth-dimensional space is the standard Euclidean space. Therefore, the fourth variable should have the same property as them. Conversely, physical dimensions are isolated figures. Four-dimensional spacetime is called Minkowski space, Minkowski who is the teacher of Einstein said four-dimensional space is the combination of three-dimensional space and one-dimensional time, which is supported by general relativity. In this case, time can be represented by a coordinate system, and it can change the order of time and space. That is the reason why there is an argument about time belonging to dimensions. N-dimensional space rotates and folds to obtain n+1-dimensional space. For example, it folds the line to become a circle which is a two-dimensional plane, and a plane is folded to form a Mobius trip. According to this method, three-dimensional space is folded to create a new four-dimensional space that might have real Kleinsche Flache.

Another method for understanding low-dimensional space is through the projection or section of high-dimensional space. A one-dimensional line can be viewed as a cut, revealing its section; a section of a two-dimensional plane is a line; for three-dimensional space, such as a cube, its section is a plane. Consequently, a cube serves as a section of a four-dimensional object known as a tesseract. In this context, the cube is comprised of six planes, while the four-dimensional space consists of eight cubes, with the cube itself being its section.

As previously mentioned, n-dimensional space is defined as a space in which n lines intersect at a common point. For four-dimensional space, this involves four lines that are perpendicular at a single point. It may be confusing to conceptualize the tesseract, particularly because a line appears not to be perpendicular to the others. However, each line must be indeed perpendicular to the point in four-dimensional space; the depiction is merely a projection that does not fully capture the complexity of four-dimensional objects.

This is analogous to representing a cube in a two-dimensional plane, where dashed lines may be employed to indicate the hidden portions or use visual art to create optical illusions. Strictly speaking, the image perceived by the human eye is merely one section of a three-dimensional object (Figure 5). Consequently, all three-dimensional space can be considered two-dimensional from a certain perspective. For instance, when envisioning a cube, the mind typically forms a rotating image, often focusing on a single face of the cube, which is determined by the viewer's perspective. If humans possessed true three-dimensional vision, they would perceive all six faces of the cube simultaneously, akin to its unfolded pattern.

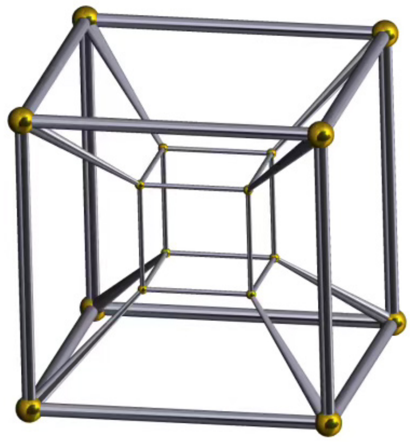

Fig. 5 The projection of tesseract (Photo/ Picture credit: Original).

The second theory pertains to the transformation into four-dimensional spacetime, wherein time is considered the fourth dimension. In this framework, a Cartesian coordinate system is employed, consisting of three axes: x, y, and z. Currently, time is represented solely along the x-axis; however, if time were to be assigned the y-axis, it would facilitate a new understanding of possibilities.

In this context, defining the y-axis as representing potentiality implies that each parallel line to the x-axis corresponds to a different possibility. Within these possibilities, an individual may exist as a student, teacher, or chef, thereby giving rise to the concept of parallel universes. According to this theory, the highest dimensions of the universe are proposed to be ten, encompassing all entities and all possibilities.

3.2 How to Enter Four-Dimensional Spaces

According to the theory of relativity, the mass of objects will change when their speed is close to the light. Thus, humans can enter the fourth-dimensional space which has a lot of perpendicular three-dimensional spaces when people's speed approaches that of light. In the space, cross mass is relatively free. Usually, some crowded parts of the four-dimensional spaces are made up of the experiences and things of someone. Some impossible things might be done in four-dimensional spaces, like time travelling. However, if humans are able to do these, they will be a higher-dimensional organism rather than before. Meanwhile, by entering higher-dimensional spaces, they perhaps abandon their body, but their awareness will remain.

3.3 Application of Dimensional Spaces

The applications of dimensional spaces mainly focus on dimensional analysis which analyses the relations between different physical quantities by recognizing their base quantities and units of measurement. Dimensional analysis is usually applied in chemistry, mathematics, and physics. Also, dimensional spaces can be applied in other fields. An example is the calculation of the volume of N-ball and the surface area. For the n-sphere, the volume scales as x^n , and the surface area scales as x^{n-1} . Hence, the volume of the n-ball in terms of the radius is c_{nr^n}, if assume the constant is represented as c_n . To confirm the constant, one should overcome a lot of difficulties, while the dimensional analysis can reduce the difficulty of calculations. In the fields of finance, economics, and accounting, dimensional analysis is normally adopted to distinguish stocks from flows. Generally, this method explains financial ratios, economic ratios, and accounting ratios. For example, the PE ratio has dimensions of time, which means years of earning to earn the price paid. In fluid mechanics, dimensional analysis is performed to obtain dimensionless pi terms or groups. According to the principles of dimensional analysis, any prototype can be described by a series of terms or groups that describe the behavior of the system. Using suitable pi terms or groups, it is possible to develop a similar set of pi terms for a model that has the same dimensional relationships.

4. Conclusion

The essay illustrated the structure of confirmed dimensional spaces and gave a mathematical example. Next, it introduced the features of zero-to-four-dimensional spaces. It mainly explored the four-dimensional spaces by mathematics and especially by physics and illustrated different kinds of physical theories and approaches because compared to mathematical ones, the physical ones are closer to real-life. Finally, the essay built on the relations of dimensional spaces and their applications.

As can be seen from the article, abstract shapes (points, lines, planes, cubes) do not exist in mathematics but are concepts constructed by mathematics. Therefore, researchers are more inclined to approach dimensional space from

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the perspective of physics, because time and possibility are more acceptable to people. In physics, dimensional spaces are often explained by fantastic analogies, such as the concept of wrinkled space. This method makes the understanding of high-dimensional space more intuitive and visual.

The dimension of the universe and the awareness are not determined, and researchers are suggested to check and analyze the structure of them. For the mentioned theories, the understanding of general relativity is indispensable, which is inspired by Riemannian geometry, and it explains or helps to understand examples, such as wormholes, parallel universes, and observers.

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