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Design of an Analog Computer for Handling Ordinary Differential Equations

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Abstract:

This study describes the design of an operational amplifier-based analog computer that can be used to process ordinary differential equations. It can be used to handle operations such as summation, negation, and integration. The study is organized as follows: Section 2 introduces the principle of circuit blocks. Section 3 introduces the Circuit design for first-order differential equation. Section 4 introduces the Circuit design for second-order differential equation. Section 5 summarizes the applications of analog computer.

Keywords: Ordinary Differential Equations; Analog Computer; Circuit design; industrial applications

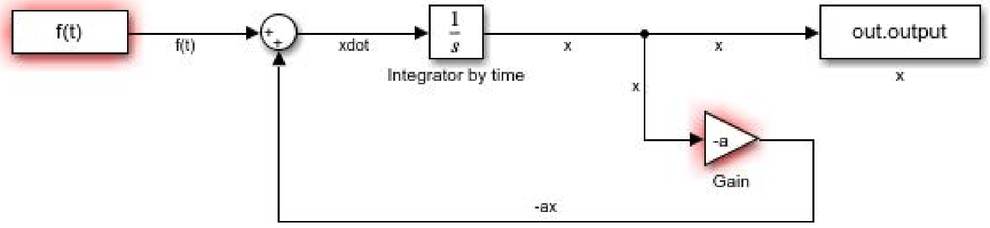
1 Introduction

An analog computer is a type of computer that uses the continuous variation aspect of physical phenomena such as electrical, mechanical, or hydraulic quantities (analog signals) to model the problem being solved[1]. Analog computers were widely used in scientific and industrial applications even after the advent of digital computers. Analog computers remained in use in some specific applications, such as aircraft flight simulators, the flight computer in aircraft, and for teaching control systems in universities. Analog computers monitor the continuously changeable entities and process a variety of calculations to display data based on the needs.

2 Principle of circuit blocks

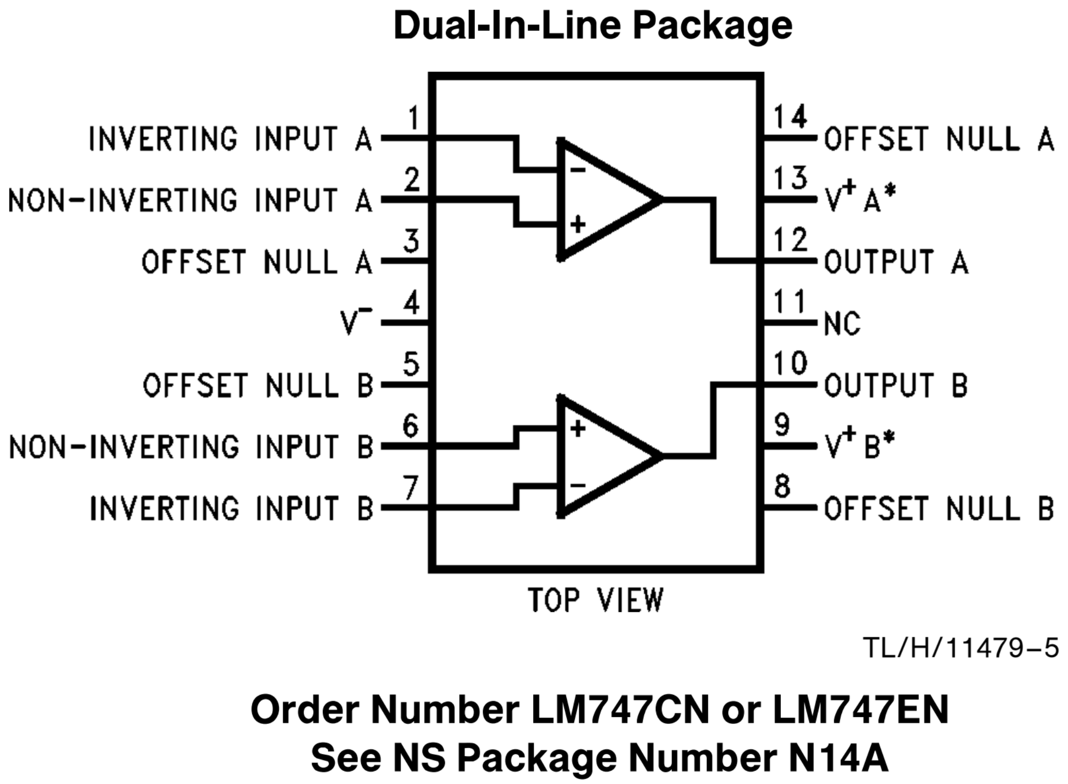
An operational amplifier relates the output-to-ground potential to the input potential difference. By arranging appropriate components, various circuit blocks such as summation, inversion, and integration can be generated.

The 1st order ODEs have a general form of equation: $\frac{dx}{dt}+ax=f(t)$. To solve the general solution of such systems, the following simulation diagram is constructed via Matlab:

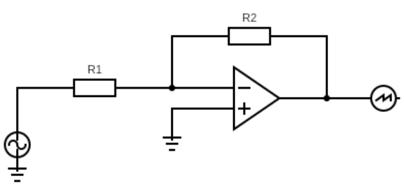


The input here is the non-homogenous forcing fuction f(t) to the left and the monitors (e.g. oscillo-

scopes) are set up along the circuit to reveal the states of the dependent variable x(t) and its derivative, xdot(t).
In this system, the modules needed include adders, amplifiers, and integrators. These blocks can be implemented with a combination of operational amplifiers and resistors and capacitors. TI's LM747 is a commonly used operational amplifier that can be used to implement this design. The logic function of LM747 is as follows[2].

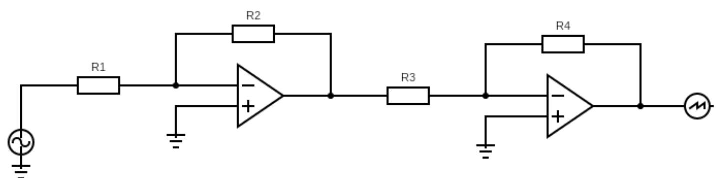


Based on LM747, an inverting amplifier can be built as shown in the figure below.



In the figure, $V_{out} = -\frac{R_2}{R_1} V_{input}$.

Taking it a step further, based on the LM747, an amplifier can be built as shown in the figure below.



An integrator can be built as shown in the figure below.

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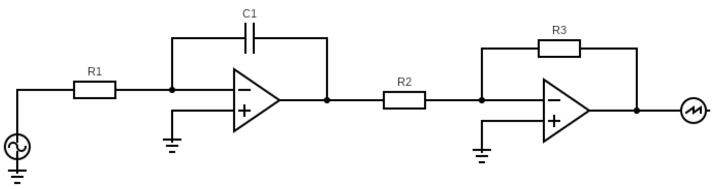
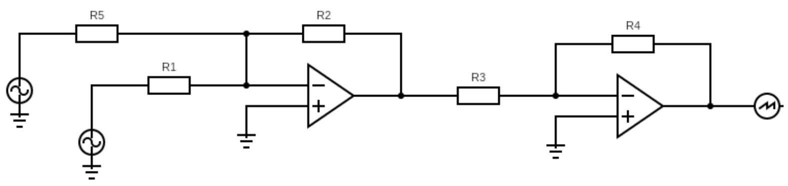


In the figure, $V_{out} = \int V_{input} dt$.

Based on LM747, an adder can be built, as shown in the figure below.



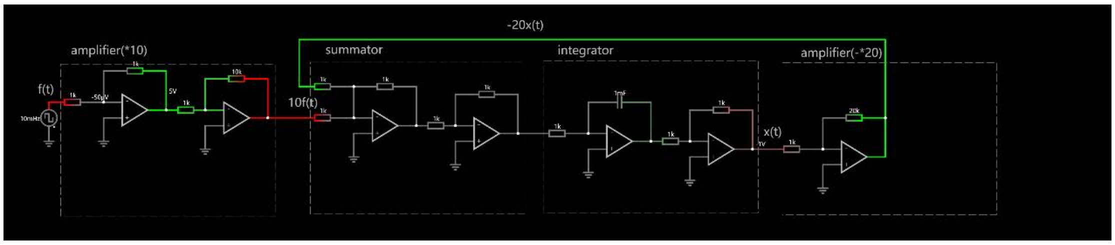
In the figure, $V_{out} = V_1 + V_2$.

3 Circuit design for first -order differential equation

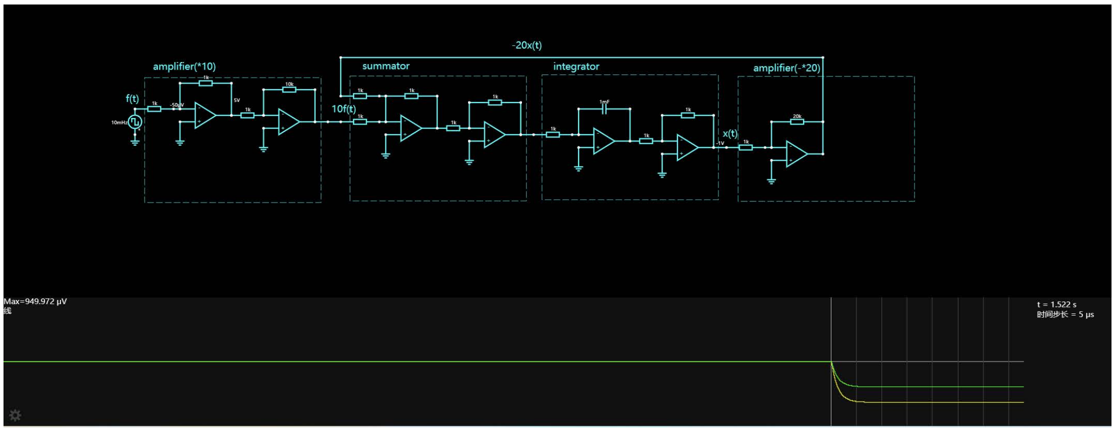
Based on the above analysis, the following method can be used to construct an analog computer to solve the first-order differential equation.

Taking $\frac{dx}{dt} + 10x = 10f(t)$ as an example, the equation can be rewritten to $\frac{dx}{dt} = 10f(t) - 10x$.

Build the circuit on Falstad as shown below.



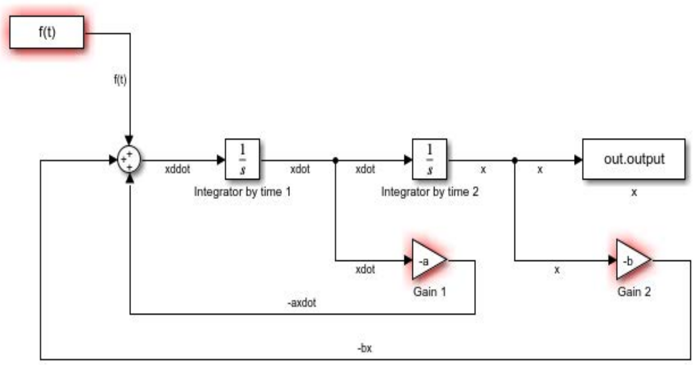
An AC square wave generator with amplitude of 1V and frequency of 250mHz is modeled as f(t). An amplifier circuit with gain of 10 is employed to result 10f(t). The following summator circuit is employed to process the summation operation. The following integrator circuit is employed to integrate xdot to x(t). The following amplifier circuit with gain of -20 is employed to result -20x(t). An oscilloscope placed after the integrator is employed to monitor the performance of the ODE solving circuit by their potential difference, which is shown below:



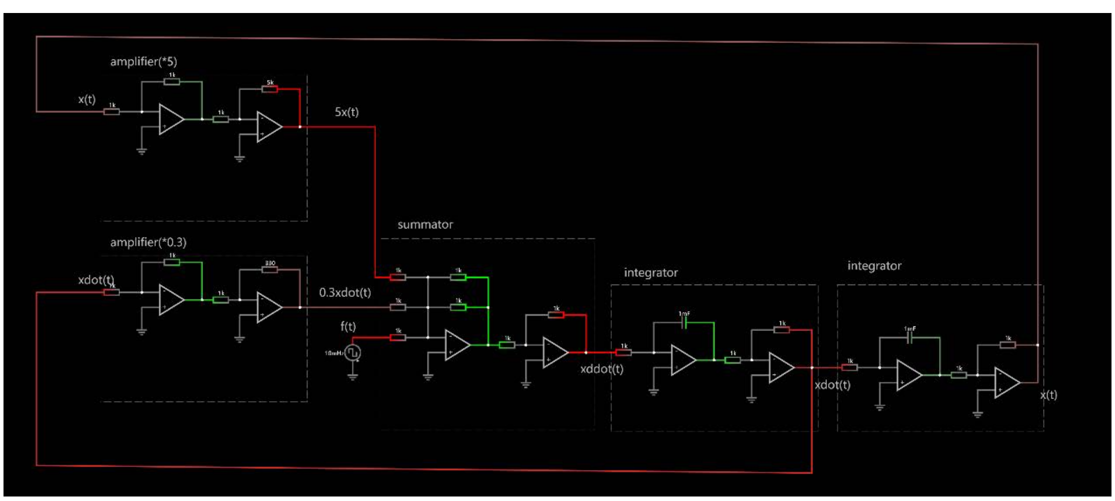
Solving first-order differential equations with different coefficients can be achieved by changing the resistance values of the resistors and bypassing some inverters.

4 Circuit design for second -order differential equation

Going a step further, it is possible to build analog computers that solve second-order differential equations.
The general equation form of 2nd order ODEs is: $\frac{d^2x}{dt^2}+a\frac{dx}{dt}+bx=f(t)$. Two integrators are employed to obtain the linear terms axdot and bx respectively. The following simulation diagram is constructed via Matlab:



Taking $\frac{d^2x}{dt^2}-0.3\frac{dx}{dt}-5x=f(t)$ as an example, the equation can be rewritten to $\frac{d^2x}{dt^2}=f(t)+0.3\frac{dx}{dt}+5x$.
Build the circuit on Falstad as shown below.



Solving second-order differential equations with different coefficients can be achieved by changing the resistance value of the resistors and bypassing some inverters.

5 Summary

The two examples above reveal some of the basic characteristics of a simple analog operator. Although digital computers are in wide application in life with the advantageous signal storage and processing capability, analog computers are still in charge of simulations and controls in aircrafts and industrial processes where reliability and reaction speed are the first concerns.

References

[1] Analog Computer Explained: Everything You Need To Know, https://history-computer.com/analog-computers/
[2] LM747 Dual Operational Amplifier datasheet, https://www.ti.com/lit/ds/symlink/lm747.pdf