

# Inflationary Epoch and CMB Observations: Bridging Theory and Experiment

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## Abstract:

This study discusses inflationary models supported by Cosmic Microwave Background (CMB) measurements from Wilkinson Microwave Anisotropy Probe (WMAP), such as the Lambda Cold Dark Matter model, also referred to be the modern standard model of cosmology. The prevailing Big Bang model is complemented by the inflationary model, which solves monopole, flatness, and horizon paradoxes and makes important predictions about the geometry of the universe and the Gaussianity of the primordial spectrum of density fluctuations. The goal of WMAP is to measure key parameters including the Hubble constant, cosmological density parameter, anisotropy spectral index, and others in order to investigate the structure, evolution, and composition of the universe. The inflationary concept, especially the  $\Lambda$ CDM model, is further strengthened by the CMB data from WMAP, which agrees well with it.

**Keywords:** Inflationary model; CMB; WMAP.

## 1. Introduction

The widely-embraced model of cosmology contemporarily is the  $\Lambda$ CDM model, which explains how the universe has evolved. The cosmological constant, denoted by  $\Lambda$ , was first presented in Einstein's General Relativity and subsequently utilized to represent vacuum energy. The mass-energy density of the universe is dominated by dark components, such as dark energy and dark matter, according to the cosmological principle that the cosmos is homogeneous and isotropic on vast scales. Based on Planck satellite data, the fraction of the universe's mass-energy density that can be assigned to dark energy is  $0.6847 \pm 0.0073$  [1]. The cosmos is expanding faster than ever before, and dark energy is mostly responsible for this, as the cosmological constant  $\Lambda$  explains. Dark

matter does not emit, absorb, or reflect light; instead, it is best explained as the result of some unidentified, non-luminous kind of matter interacting with normal matter via gravity but not electromagnetic forces. This is included in the model of  $\Lambda$ CDM. Conversely, about 27% of the mass-energy density of the universe is made up of dark matter. It is an invisible, non-luminous kind of matter that does not interact with electromagnetic forces; yet, its presence is implied by the gravitational pull it exerts on cosmic structures and galaxies.

A key element of the  $\Lambda$ CDM model is dark matter, along with dark energy [2]. According to the  $\Lambda$ CDM model, the cosmos is flat, which means that the total energy density is equal to the critical density needed to maintain the universe's geometrical flatness. Cur-

rent observational data from sources such as the CMB is compatible with this.

The Big Bang, the starting point of the  $\Lambda$ CDM model, was a solitary event that involved the rapid appearance of expanding spacetime that was teeming with high-energy radiation at a temperature of approximately  $10^{15}$  K, rather than an explosion. The cosmos had a period of fast exponential expansion immediately after this event, known as cosmic inflation, which caused the universe's size to increase by a factor of  $10^{27}$  or more.

The universe stayed extraordinarily hot (over 10,000 K) for some hundred thousand years after inflation. This early state of the universe is observable today from CMB, a faint, low-energy radiation detectable from all directions in the sky. The universe is still expanding, there are a lot of light elements (helium, lithium, and hydrogen) in the universe, and the CMB has a detailed structure and anisotropies that reflect the tiny density fluctuations in the early universe. These observations can only be consistently explained by the Big Bang theory when combined with cosmic inflation and standard particle physics. As a result, the most effective and generally acknowledged theory explaining the evolution of the universe is the  $\Lambda$ CDM model.

## 2. Theoretical Framework

The field equations, which form the basis of Einstein's general relativity, are the source of the Standard Big Bang concept:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi GT_{\mu\nu}. \quad (1)$$

A symmetry in the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric tensor arises from the cosmological principle's assumption notion the universe is isotropic and homogeneous in accordance with observations:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \quad (2)$$

where  $ds^2$  is the spacetime interval between two adjacent events,  $dt^2$  is time interval,  $a(t) = e^{Ht}$  is the scale factor that characterizes the expansion or contraction of physical distances, and  $k$  is the curvature parameter. A flat world is represented by  $k=0$ , a closed universe by  $k=1$ , and an open universe by  $k=-1$ . The spatial part of the FLRW metric describes a uniform universe, where distances between objects only change due to the scale factor  $a(t)$  but not due to any variation in matter distribution. This uniformity holds at large scales, even though at smaller scales, structures like galaxies and clusters exist. The FLRW metric's

dependence on the spherical coordinates is symmetric.

The  $r^2(d\theta^2 + \sin^2\theta d\phi^2)$  terms imply that the spatial geometry is the same in all directions, which embodies isotropy.

Complemented by FLRW metric, the field equation becomes

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \quad (3)$$

where the Hubble parameter,  $H$ , indicates how quickly the cosmos is expanding, while  $\rho$  represents the energy density of the universe, which takes into account contributions from dark energy, matter, and radiation.  $G$  is Newton's gravitational constant, so the  $\frac{8\pi G}{3}\rho$  and  $-\frac{k}{a^2}$  terms relate to energy density and spatial curvature of the universe.

Even while the Big Bang Theory is compatible with measurements of the Cosmic Microwave Background (CMB), other issues remain unsolved, such as the issues of monopole, horizon, and flatness. Thankfully, there are workable answers to these issues in inflationary theory, which postulates an era of exponential universe expansion preceding the Big Bang and hence enhances the Big Bang Theory [3]. The furthest radiation in the microwave range that can be observed by any telescope was produced immediately after the Big Bang and is known as CMB. In addition to providing evidence for the Big Bang Theory, whereas the CMB's existence is explained by the Big Bang hypothesis. The isotropic CMB implies that two very far points of the universe share similar properties such as temperature, urging the advent of inflationary theory to legitimate the CMB observation.

The Wilkinson Microwave Anisotropy Probe (WMAP) was one of the successful CMB measurement programs. Its main objective was to examine the CMB's anisotropies throughout the sky and offer comprehensive details on the age, composition, and evolution of the early universe. WMAP has yielded numerous important findings. First, it is estimated that the universe is 13.8 billion years old. Second, the universe contains 71.4% dark energy, 24% dark matter, and 4.6% regular matter. Finally, and perhaps most crucially, the geometry of the universe is flat.

## 3. Observational Techniques

NASA launched the WMAP satellite on June 30, 2001, with the goal of creating a complete sky map of the CMB and researching the characteristics of the cosmos [5]. Specifically, WMAP measures a number of intriguing properties in an effort to better understand the universe's

composition, shape, and development, such as the anisotropy spectral index, cosmological density parameter, the Hubble constant, etc.

The main difficulty of WMAP mission is to accurately and precisely measure the temperature of two points in the universe that form an angle of 180 degrees. At the same time, measurements are required to be as independent as possible to allow for statistical analyses with lists of temperature and weight.

The two Gregorian telescopes that make up WMAP concentrate radiation onto 10 pairs of back-to-back feeds, that accept radiation frequency between 20 to 100 GHz. Then, an orthomode transducer would produce two linear orthogonal polarizations, each of which would be amplified and detected subsequently [6].

The Science and Mission Operations Center (SMOC) receives raw data from the satellite. SMOC mainly retransmit missing data, put data in time order, and conduct health and safety checks. After checking, the data are used to plot a set of daily graphs to be inspected and reduced “trending archive” consisting of data sampled every ten minutes. As data are inspected for pre-set range limits and time-gradients, a person of responsibility will be notified of unqualified data sets. Then, procedures of generating archive collate data files, interpolating attitude, and flagging suspect data and planet data would follow. Sky maps are solved after calibration [6].

The WMAP measured telescope beams from observations of Jupiter. Each year two observation seasons lasting about 50 days occur. During the WMAP nine-year mission, 17 seasons of Jupiter observations were acquired. Since Jupiter’s brightness and angular size are well-known, it is easier to calibrate the telescope. Since Jupiter is a bright point-like source, it provides a consistent and well-defined light signal, allowing for measurements the telescope beam’s shape, size, and response. These observations help calibrate the telescope, correcting systematic errors, and ensuring the precision of CMB data [7]. Since the optical properties of WMAP’s A and B telescopes differ slightly, Jupiter and the CMB do not appear at the same temperature when measured by each side. This imbalance, around 1%, is corrected in the sky maps produced in the mission’s first year. Beam transfer functions, which describe the beam’s response in spherical harmonic space, offer a thorough description of how beams interact with the CMB data. This reaction is quantified using a coordinate system that is fixed to the WMAP satellite. Beam transfer function computation involves a number of steps. One feed at a time, observations of Jupiter are obtained for each differential assembly. Separate processing is done on the data for the A- and B-side beams to account for the static sky background. Following correction, a planar grid

with each of the 20 A- and B-side boresights at its center is created using the data [7].

## 4. Bridging Theory and Experiment

The Gaussian primordial spectrum of density fluctuations and the cosmic density parameter  $\Omega$  around 1 are predicted by inflationary theories [4]. The ratio of the density derived from observation to that computed from a flat universe is known as the cosmological density parameter. Thus, the flatness of the cosmos is implied by the inflationary hypothesis. In addition, a roughly scale-invariant gravitational wave spectrum is known to exist, and it is anticipated that the primordial power spectrum’s slope would differ from 1.

To discriminate inflationary models, precise measurements of primordial fluctuations are essential—a signal of non-Gaussianity and CMB anisotropy are reliable tests for inflationary models. Specifically, the WMAP data disfavor the exact Harrison-Zel’dovich-Peebles spectrum, indicating existence of gravitational waves. However, there are inflationary models that suggest small amplitude and even nondetectable gravitational waves. Gaussianity tests are crucial to examine inflationary models, such as slow roll, curvaton, multi-fields, etc. Inflation in standard slow roll model barely produces non-Gaussianity, which stems from second order perturbations after inflation. More non-Gaussianity could be produced by other models [9]. Gaussian primordial fluctuation could be confirmed by measurements of non-Gaussianity of the CMB [7]. Two statistical methods are applied and the same conclusion in favor of Gaussianity is attained: a cubic statistic measuring phase correlations of temperature fluctuations and the Minkowski functionals measuring the morphology of sky maps. Not abundant, massive clusters of galaxies at high redshift also provide clues for primordial fluctuations. The number of these clusters could be calculated to match a Gaussian distribution. A slight non-Gaussianity can determine the shape of tails of density fluctuation distribution. Halos are more abundant at high red shift, thereby making it possible to identify non-Gaussianity more accurately [10]. The WMAP data provide evidence of primordial non-Gaussianity of local kind, which is resilient to fluctuations in maximum multipole moments and frequency. The signal is not supported by known foreground, instrument systematic, or secondary anisotropy. This hypothesis is fundamentally contradicted by WMAP 3-year data, as modest slow inflation suggests little non-Gaussianity [11]. The WMAP data provide light on the polarizations of the B- and E- modes. The two polarizations that make up the CMB are the E-mode and the B-mode. Energy density variations give rise to E-mode polarization, which is

sometimes referred to as gradient mode polarization [12]. E-mode polarizations are produced by Thomson scattering and curl free. The E-mode is closely linked to the temperature anisotropies of the CMB because it is produced by density disturbances. In the cosmos, e-modes are tangential around hot areas and radial around cold spots. Weaker in amplitude than E-mode, B-mode polarization stems from tensor perturbations due to propagation of gravitational waves within CMB, which entitles B-mode measurements great importance in the study of inflationary theory. The divergence free B-mode cannot be created by scalar but tensor perturbations. Polarization signals could be used to free electrons because the CMB anisotropies are only polarized by Thomson scattering, which is defined as elastic scattering of electromagnetic radiation that accelerates a charged particle and emits an electromagnetic wave with the same wavelength but a different direction from the incident wave. Additionally, polarization signals help explain sources of temperature anisotropies as they give rise to different polarization patterns. Beyond B-mode tests for inflationary theory, non-Gaussianity of the scalar spectrum could also enrich physical processes during the inflation [13].

## 5. Results

According to the first year of WMAP data, the primordial fluctuations are Gaussian and the universe is flat [14]. The discovery by WMAP of an anti-correlation between CMB temperature and polarization variations is compatible with the inflationary paradigm. Two field inflationary models are tested, but the results disfavor the additional parameters [8].

The inflationary flat  $\Lambda$ CDM model is accepted as the mainstream model of cosmology because it corresponds well with nine-year WMAP data. The universe's early circumstances, including the creation of primordial disturbances, are provided by inflation. It is believed that the inflaton's quantum fluctuations are the source of these disruptions. a hypothetical field driving inflation, and gravitational fields. The flatness parameter  $\Omega_k$  quantifies the deviation from a flat geometry. The findings of nine-year data indicate that  $\Omega_k = -0.0031 \pm 0.0038$  and  $|\Omega_k| < 0.0094$  at a 95% confidence level. If the universe is assumed to have a negative curvature open universe, the data restricts  $\Omega_k$  to be less than 0.0062 at 95% confidence level. The observed properties of CMB fluctuations, including their adiabatic nature and Gaussian random phases, strongly support inflation. The CMB observations have been crucial in posing significant cosmological prob-

lems (horizon, flatness, and structure problems) that inflation helps resolve within the framework of general relativity. Essentially, these results provide more evidence in favor of inflation as the primary explanation describing the dynamics of the early cosmos as well as the creation of cosmic structures [15].

## 6. Summary

The Big Bang Model reconciles evidence like the cosmic microwave background (CMB) and explains the universe's development. The inflationary theories provide an exponential growth period around  $10^{-36}$  seconds after the Big Bang, which addresses the flatness, horizon, and monopole issues with the Big Bang Model. The CMB data from Wilkinson Microwave Anisotropy Probe (WMAP) measurements fits with the inflationary theories. In particular, the  $\Lambda$ CDM model agrees well with nine-year WMAP data, approved as the dominant cosmological model. The results obtained from WMAP were crucial in comprehending the origins and large-scale structure of the universe, and they also contributed to the improvement of the  $\Lambda$ CDM model. Among the major findings of WMAP are the following: the universe is composed of around 4.6% normal matter, 24% dark matter, and that 71.4% dark energy; the universe is geometrically flat with the flatness parameter  $\Omega_k = -0.0031 \pm 0.003$  and  $|\Omega_k| < 0.0094$  at a 95% confidence level. WMAP observations are conducted using two Gregorian telescopes placed back-to-back on which radiation is focused to ten feed pairs that receive radiation at frequencies ranging from 20 to 100 GHz. Raw data are processed by SMOC including retransmitting missing data, sorting data in time order, and conducting health and safety checks. Finally, sky maps are solved after calibration. As a well-defined light signal due to known brightness and point-like feature, the planet Jupiter is utilized for calibration of beam shape, size, and response of the telescopes, which facilitates precision of CMB data.

The inflationary models have several major predictions: the cosmological density parameter  $\Omega$  is nearly 1 (a flat universe); primordial spectrum of density fluctuations is Gaussian; there is a virtually scale-invariant gravitational wave spectrum. These predictions are confirmed by the WMAP. The WMAP first year data conclude that the universe is flat. The WMAP data specifically points to the presence of gravitational waves and contradicts the spectrum of Harrison, Zel'dovich, and Peebles. As for Gaussianity, precise measurements of primordial fluctuations are necessary to distinguish inflationary models. Gaussian

primordial fluctuation could be confirmed by measurements of non-Gaussianity of the CMB: two statistical methods are applied and the same conclusion in favor of Gaussianity is attained. Furthermore, the quantity of large galaxy clusters at high redshift corresponds with a Gaussian distribution. Slow roll inflation that indicates negligible non-Gaussianity is disfavored by WMAP 3-year data. The CMB may be divided into E-mode and B-mode polarizations. Tensor perturbations caused by the passage of gravitational waves inside the CMB induce B-mode polarization, whose measurements play a role in the study of inflationary theory. The inflationary theory is compatible with WMAP's finding of an anti-correlation between CMB temperature and polarization variations. The observed properties of CMB fluctuations, including their adiabatic nature and Gaussian random phases, strongly support inflation.

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