Dean&Francis

ISSN 2959-6157

The Application of Definite Integral In Different Aspects

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Abstract:

A definite integral is a form of a calculation of area beneath of function, using infinitesimal silver or stripes of the region. Different from indefinite integral which representing a function, definite integral is a specific value. The value of a definite integral only depends on the interval of this integral, which means for the same function and interval, the result won't change with different variables. Integrals may represent the area of a region, the accumulated value of a function changing over time, or the quantity of an item given its density. This method can be used to solve many problems in different fields. This paper will explore some applications of definite integral. This article first introduces the applications of definite integrals in geometry, and then studies their applications in physics, such as variable force work and pumping work. Finally, how to use the properties of definite integrals to solve limit problems will be further studied.

Keywords: Geometry; Physics; limit

1. Introduction

The applications of definite integral are very useful. In geometric field, the definite integral can be used to find the area under curve. Definite integral also has wide applications in physics, for instance, calculating the displacement of an object moving with variable velocity, the work, pressure and density produced by changing force. In the field of engineering, definite integrals are used to calculate the volume, surface area, and other geometric quantities of objects, which is crucial for design and analysis in areas such as architectural design and mechanical manufacturing.

In 2007, Wang did research about the applications of definite integral about area, volume, distance and some problems in life [1]. In 2008, Xin wrote some applications in geometry and economics [2]. In 2009, Wang provided examples to illustrate the simple application of definite integrals in economics [3]. In 2012, Zhang introduced the applications by using the Power Plant Thermal Power Equipment Specialty as the background [4]. In 2018, Yang used definite integral to solve problems about doing work with changing force [5]. In 2023, Liang, Wang and Li studied the application of Monte Carlo method in solving definite integrals [6]. In 2003, Wang found out a common mistake in applications of definite integral [7]. In 2004, Wang discussed the application of definite integrals in the form of $\theta=\theta$ (r) for curve equations [8]. In 2004, Luo and Yang analyzed an error problem that is prone to occur in the application of definite integrals [9].

This paper will discuss the main applications of definite integral. The section 2.1 is about the geometric applications of definite integral and some examples. The section 2.2 is about the physics applications of definite integral and some examples. The section 2.3 is an example of using definite integral to find out the limit of a function.

2. Applications of Definite Integral

2.1 Geometric Applications of Definite Integral

The definite integral is the sum of the height of the function and the product of the length of the intervals, the height being integrated with the interval that includes the formula of the area of the rectangle.

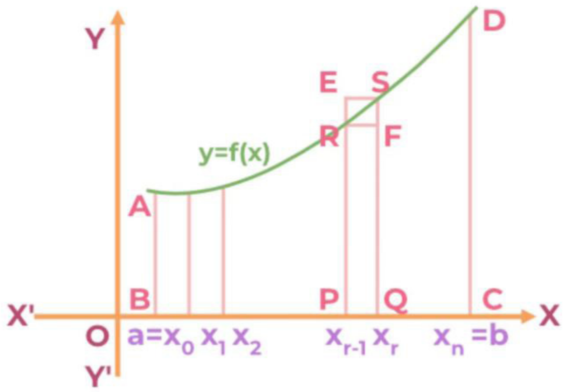

Fig. 1 Area segmentation

First, insert several points in the interval [a, b] shows as Fig. 1, where a=$x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n =$b. Divide the interval [a, b] into several smaller interval, then the length of each subinterval is $\Delta x_r = x_r$ - x_{r-1} (r=1,2,3,…,n). Here we use infinitesimal method.

To find out the area of the trapezoid with cured side, the shaded area can be divided into many rectangles in the Fig. 1. Thus, the area of the shaded area is the sum of the area of these small rectangles. α_1 is the midpoint between x_0 and x_1, so f (α_1) is the height of the first rectangle. Then, the area of the first rectangle $S_1 \approx$ f (α_1)$\bullet \Delta x_1$, S_2, $S_3 \ldots S_n$ are similar to S_1. Thus, the total area S $\approx S_1 + S_2 + S_3 + \ldots + S_n \approx \sum_{r=1}^{n} f(\alpha_r) \bullet \Delta x_r$.

λ representes the maximum value of Δx_i , when $\lambda \to$ 0, $\Delta x_i \to 0, n \to \infty$,so,thelimitexists.

Then the area S= $\lim_{\lambda \to 0} \sum_{r=1}^{n} f(\alpha_r) \bullet ?x_r = \int_a^b f(x) dx.$

According to this property of the definite integral, the definite integral can be applied to some geometric questions. There are three main categories of geometric applications: the area of two-dimensional figure, volume of rotated solid and arc length of plane curve. Here are some examples.

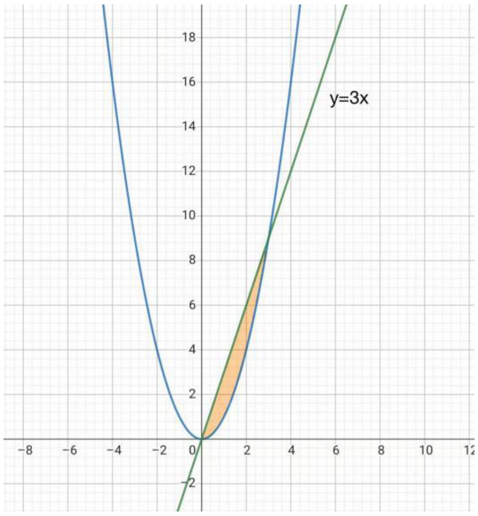

Fig. 2 The graph of two functions

Example 1. Find out the area bounded by the curve y= x^2 and the line y=3x.

Proof:

According to the Fig. 2, $\begin{cases} y = 3x \\ y = x^2 \end{cases}$ leads to the intersection points are (0,0) and (3,9). Thus, the interval is [0,3]. Area of R= $\int_0^3 3x - x^2 dx = [\frac{3x^2}{2} - \frac{1}{3}x^3]_0^3 = \frac{9}{2}$.

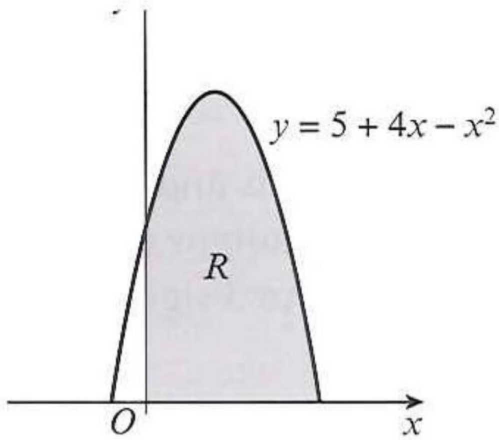

Fig. 3 The area bounded by some lines.

Example 2: The curve shown in the Fig. 3 is y= $-x^2 + 4x$ +5. The finite region R is bounded by the curve, the x-axis and the y-axis. The region is rotated through 2 π radians about x-axis to generate a solid of revolution. Find the exact volume of the solid generated.

Proof:

According to the formulae Volume= $\pi \int_a^b y^2 dx$, a=0, b=5, so V= $\pi \int_0^5 (5 + 4x - x^2)^2 dx = 250\pi$.

2.2 Physics Applications of Definite Integral

The infinitesimal method can also be used in physics field.

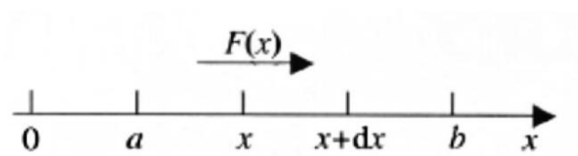

Fig. 4 The distance travelled by the force.

Example 3:

An object is moving from a to b under a changing force F(x), so the work by the object can be found out by using infinitesimal method. Because the change of force is continuous, so F(x) is a continuous function.

In the Fig. 4, set up a coordinate system. Divided the interval [a, b] into many small parts, and consider the force act on the object (F(x)) during each small part is constant. Then, the work of object in the small interval can be calculated by using work=force times distance. Finally, by using infinitesimal method, the total work the object did is W= $\int_a^b F(x) dx$.

Example 5:

Another example about infinitesimal method in physics is the work when pumping the water. We need to find out the work needed to pump out all the water in this cone. The radius r of the base is 4m.

Proof:

First, set up a coordinate system in the Fig. 5, where x is the distance from water to the base of the cone. x is the variable of the integration, the interval of the integral is [2, 10]. In this interval, we take a small interval and the volume of the corresponding cylinder is represented as dV= $\pi r^2 \bullet ? x = \pi \bullet \frac{4}{25} \bullet (10 - x)^2 \bullet dx$. So, the force used dF=dm $\bullet g = \rho g \bullet dV = \rho g \pi \bullet \frac{4}{25} \bullet (10 - x)^2 \bullet dx$. Thus, dW=dF $\bullet x = \rho g \pi \bullet \frac{4}{25} \bullet (10 - x)^2 \bullet dx \bullet x$.

Finally, the total work W= $\int_2^{10} \rho g\pi \bullet \frac{4}{25} x \bullet (10-x)^2 \bullet dx$ which also use definite integral.

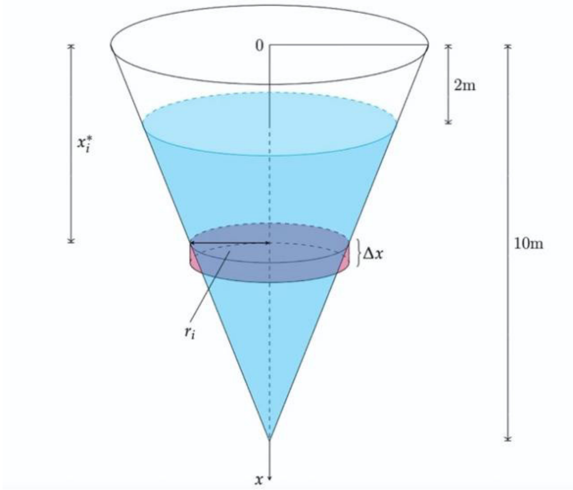

Fig. 5 Pumping out the water from the cone.

2.3 Application of Definite Integral in Limit

The definite integral can represent the limit of a sum of n terms, namely

$$\lim_{n\to\infty}\sum_{r=1}^{n} f(\alpha_r)\bullet ?x_r = \int_a^b f(x)dx.$$

Because the limit on the left-hand side has no connection with [a, b] and the value of α_r, so assume the interval is [0, 1] and divide the interval equally, then we get the length of the subinterval $\Delta x_r = \frac{1}{n}$, so the midpoint of the midpoint $\Delta x_r is \frac{r}{n}$. Thus, the equation (1) can be written as

$$\lim_{n\to\infty}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)\bullet\frac{1}{n} = \int_0^1 f(x)dx. \quad (2)$$

The equation (2) can be used to find out the limit of a sum of n terms.

Example 6:

Find out the limit of $\lim_{n\to\infty}\frac{1}{n}[(n+1)(n+2)\ldots(n+n)]^{\frac{1}{n}}[10]$.

Proof:

Because the formula is in the product form, so we consider solving this problem by rewriting the formula in the logarithmic form.

$$\frac{1}{n}[(n+1)(n+2)\ldots(n+n)]^{\frac{1}{n}} = (\frac{1}{n})^{n\bullet\frac{1}{n}}\bullet[(n+1)(n+2)\ldots(n+n)]^{\frac{1}{n}}$$

$$=[\frac{n+1}{n}\bullet\frac{n+2}{n}\bullet\ldots\bullet\frac{n+n}{n}]^{\frac{1}{n}} =[(1+\frac{1}{n})(1+\frac{2}{n})\ldots(1+\frac{n}{n})]^{\frac{1}{n}}. \quad (3)$$

Then, transforming the formula in logarithmic form leads to that

$$\lim_{n\to\infty}[(1+\frac{1}{n})\left(1+\frac{2}{n}\right)\ldots(1+\frac{n}{n})]^{\frac{1}{n}}$$

$$=\lim_{n\to\infty}\frac{1}{n}\bullet ln[(1+\frac{1}{n})\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{n}{n}\right) = \lim_{n\to\infty}\frac{1}{n}\bullet[ln(1+\frac{1}{n})+ln(1+\frac{2}{n})+ln(1+\frac{n}{n})]$$

$$=\lim_{n\to\infty}\frac{1}{n}\bullet\sum_{r=1}^{n} ln\left(1+\frac{r}{n}\right) = \int_0^1 ln(1+x)dx =[(1+x)\bullet ln(1+x)+x]_0^1 = e^4 - 1. \quad (4)$$

3. Conclusion

In conclusion, the role of definite integrals in mathematical research is multifaceted, providing us with a powerful tool to better understand and solve various complex real-life problems. The mathematical structure of many problems is the same as that of finding "limit of sum" in definite integral. The concept of definite integral gradually developed and established. Definite integral can solve the curved edge trapezoid area, variable linear motion and other practical problems such as the use of limit method, the division of the whole, local linearization, direct bending, finite to infinite, continuous to discrete and other processes. It can be said that the most important function of definite integral is to provide a thinking method (or thinking mode) for us to study some basic problems.

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