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# Analysis of the Principle for Gravitational Wave Detection Based on Pulsar Timing Arrays

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**Abstract:**

As a matter of fact, the hunt for gravitational waves have been under the spotlight since LIGO researchers took home the Nobel Prize. In reality, different from the laser interferometers used by LIGO, pulsar timing arrays are also feasible ways to detect gravitational waves. With this in mind, this article will mainly focus on how pulsar timing arrays could detect gravitational waves as well as its achievements so far. To be specific, pulsar timing arrays consisted of rotating pulsars in space, and their regular burst of radio waves serve as stopwatches. When gravitational waves pass by, the frequency at which human beings' receiving radio waves will change at the same time. Pulsar timing array projects around the world have been busy recording pulsars up to now, but more discoveries will come out soon. They also utilized their data to verify previous calculations such as the Helling-Downs curve. According to the analysis, the current limitations and prospects are demonstrated.

**Keywords:** Gravitational waves; pulsar timing arrays; stochastic gravitational background.

## 1. Introduction

Centuries earlier, Sir Isaac Newton made the hypothesis in his Principia that space and time are "absolute", and "without relation to anything eternal" [1]. However, Einstein's Relativity theory revolutionized the understanding on these abstract conjectures. Space and time are simply values measured by respective equipment like stopwatches and rulers, and these two properties can be combined into a single concept named "Spacetime" [2]. The original Newtonian gravity field equation, as shown below, indicates that the change in force is produced instantaneously, as the time t is the same on both sides [3]:

$$\varphi_N(x,t) = -G\int \rho(y,t) r^{-1} d^3y, r \equiv |x-y| \tag{1}$$

However, special relativity tells that the speed at which any information travels should not exceed the speed of light, and adjustments have to be made to refine the equation. This adjustment leads to the relativistic field of gravity:

$$\varphi_R(x,t) = -G\int \rho(y, t-r/c) r^{-1} d^3y \tag{2}$$

Here, a term is subtracted after time t which indicates that the force is felt only after a period of r/c. In order to simplify the equation, some approximations need to be implied. First, assuming looking at a place much further away, and that r is approximately equal to |x|. This also gives us an updated lapse in time of $t_0$-t=|x|/c. Numerical calculations derive the equations:

$$\varphi_R - \frac{GM}{|x|} + \frac{Gn_iP_i}{c|x|} - \frac{G}{2c^2}\frac{\ddot{I}_{ij}\,n_in_j}{|x|} \tag{3}$$

$$h - \frac{G}{2c^4}\frac{\ddot{I}_{ij}\,n_in_j}{|x|} \tag{4}$$

where h represent the magnitude of the field, $I_{ij}$ is defined as the quadruple tensor of the source:

$$I_{ij}(t) \equiv \int \rho(y,t) y_i y_j d^3y \tag{5}$$

An analogy can be made to compare the gravitational field with electromagnetic fields: electromagnetic waves are generated by moving charges, and in a similar manner gravitational waves are created by accelerated masses. The electromagnetic waves have been analyzed extensively in the past centuries. The main reason is that the gravitational wave is too small to be detected with regular equipment. When estimating the orders of magnitude of gravitational waves, one could derive the luminosity in gravitational wave of the source:

$$L = 4\pi|x|^2 F = \left(\frac{c^3}{G}\right)|x|^2 \dot{h}^2 \left(\frac{c^3}{G}\right)\omega^2 |x|^2 h^2 \tag{6}$$

To understand this equation, one looks at a very iconic pulsar named the crab pulsar, designated as PSR B0531+21. It is at the center of the crab nebulae, famous for its association with earlier supernova explosions. The pulsar is 1900 parsecs away from earth, the radiation frequency ω about 380 /s, and as one knows the luminosity should not exceed the rate in which pulsar losses energy, one could provide a rough estimation of the upper limit of the magnitude: Less than $6\times10^{-25}$ . The upper limit for magnitude for other celestial objects can be also estimated: L<= $10^{-21}$/d (distance in 10Mpcs) for supernovae for example. The paper is organized in this way: to begin with, one explains the previous detection methods in Sec. 2, and move on to the principle of pulsar timing arrays in Sec. 3. Sec. 4 will introduce the composition of a whole pulsar timing array project. In Sec. 5, this study analyzes some observation results, and one comes to the limitations and prospect of the pulsar timing array method in Sec. 6. Eventually, a brief summary is given in Sec. 7.

## 2. Basic Descriptions

One knows that charged particles oscillate in the electromagnetic waves, and intuitively gravitational waves affects objects in a similar manner. The wrinkles in time and space cause the particles to gain an extra acceleration, and this stretches adjacent particles apart. The change in separation over the original separation is the magnitude as $|\delta l_0 / l| = |h_0|$. There have been various attempts to detect gravitational waves for it is closely related to the special relativity theory. The first method to detect this intimidating tiny change is a bar proposed by Weber [4]. By measuring the elasticity energy acquired by the bar, researchers hope to detect the tiny variation in its length. To maximize the variation, the bar must have a fundamental frequency similar to the incoming wave. Other requirements such as deep freezing and isolation from noise make this method difficult to set up. Although Weber claimed to have found the gravitational waves with his aluminum cylinders, replication attempts were in vain. The most famous example of detected gravitational wave is in 2015, where the LIGO (Laser Interferometer Gravitational-Wave Observatory) and VIRGO collaboration detected gravitational wave from GW 150914, where a binary black hole system merged. This discovery granted the founders of LIGO Nobel prizes [5, 6]. LIGO and VIRGO are both Laser Interferometers and a principle is shown in Fig. 1 [5].

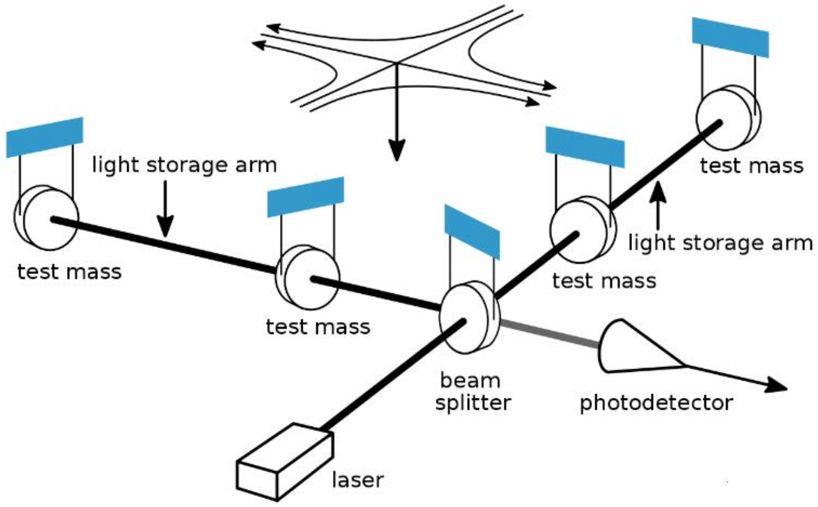

**Fig. 1 A sketch for the principle of LIGO [5].**

They are Michelson interferometers, each with two arms 3-4 kilometers long. As indicated in the graph, a beam of laser passes through a splitter, where they each pass through several kilometers on their own, returned and combined into one beam again. When gravitational waves pass through in the direction in one arm, only one of the two perpendicular arms varies in length. Based on the properties of the combined laser, the detector could determine the change in phase and also change in their respective arm length, which could be caused by gravitational waves as well as terrestrial displacement. To get rid of the glitches and noises, several interferometers are constructed around the globe. The most famous ones are two LIGOs in Washington and Louisiana, US, and one Virgo in Cascina, Italy. GW 150914 as well as many other successful observations has been cross-verified with different interferometers. After receiving raw data, it underwent several procedures such as calibration, whitening, template matching and validation before the data is cataloged. This method has been under the spotlight for a decade and has made significant achievements, as dozens of gravitational wave cases have been observed.

It is very difficult to detect gravitational waves with LIGO interferometer, for the arm length is too short. The detectable magnitude is related to the wavelength of lasers in the relationship as $h\lambda / L$. Since lasers typically have a wavelength of $10^{-6}$ m, arm lengths are around $10^{3}$ m, the magnitude is around $10^{-9}$, much smaller than required the upper limit (for the two designated black holes) of $10^{-20}$. Of course, longer laser arms could improve the sensitivity, but it is usually not feasible. The researchers derived a method to “store” the light travel time. For low frequency gravitational waves, the light could travel multiple times before the wave passes through the device. In GW150914, where the frequency is around 100, the light could make a thousand trips.

## 3. Pulsar Timing Arrays

In this paper, one will introduce a rather new method which is consisted of “pulsar timing arrays”, or PTA [7]. Pulsars rotate quickly and emit periodic wave bursts [8]. This process is more precise than clockwork, and the understanding of pulsar timing is “near the limit of models of Earth’s motion”. The observations of pulsars provide us with a celestial clock in space, as these stars have large moment of Inertia. These “lighthouses” in space could provide accurate timing without relating to clocks on earth. Moreover, pulsars can serve as point masses to check for incoming gravitational waves, as these waves are calculated to cause discrepancies between expected arrival time of pulsar bursts and the actual arrival time [9]. Although a redshift is also induced in the process, researchers are more interested in calculating the time residual.

The relationship between magnitude of the gravitational wave, residual time, and frequency of gravitational wave is shown in the approximation $Rh / f$. Traditional interferometers could only detect hundred-hertz gravitational waves. For example, the most famous GW150914 was discovered while sweeping frequency from 35 to 250 Hz. This frequency was generated by celestial objects moving relatively faster, like compact binary systems. It was discovered that two black holes moving around each other merged to emit the gravitational wave detected in this case. Pulsar timing arrays, on the other hand, were capable of detecting gravitational waves with much lower frequency, often to the magnitude of nanohertzs. The ultra-small frequency is usually caused by binaries which move at a rather small angular velocity. For pulsars with root mean square residuals (the change in time residual might imply the bypassing of gravitational waves)of around 100 ns and the target gravitational frequency of around $10^{-8}$, the magnitude of the wave must be around $10^{-14}$, much larger than the gravitational waves generated by pulsars covered in the last part. In reality, gravitational waves with such large magnitude is most likely produced by supermassive blackholes, which are often centered in the middle of galaxies. Detecting these waves not only verify the special relativity theory, but also help us understand formation of galaxies. One of the groundbreaking discoveries in the pulsar timing method is the Helling and Downs curve [10]. They applied data from several pulsars and calculated how pulsars are related to each other given their separation [11]. Fig. 2 shows that the correlation is negative when the pulsars are around 90 degrees across, and positive in small and large separations with respect to earth. This correlation is important if one wants to detect the stochastic gravitational wave background(GWB). The background emission will affect various pulsars, and by observing these affected pulsars and comparing them to the Helling-Downs curve, the researchers could search for clues on the GWB. The nanograv project also used their data on pulsars to cross-verify this relationship [12].

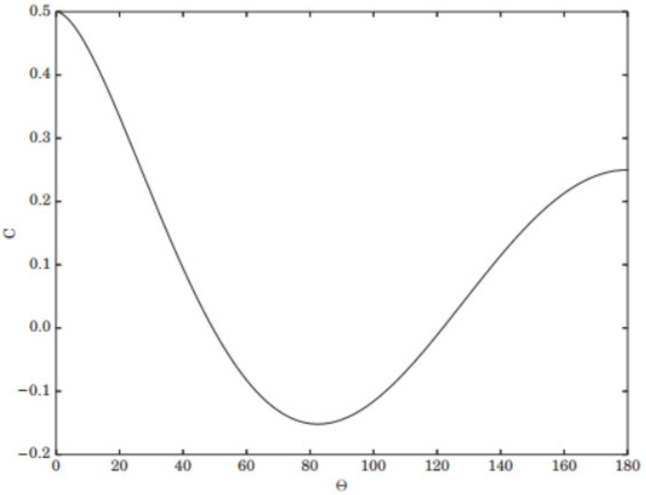

**Fig. 2 The correlation of pulsars as a function of theta [12].**

## 4. State-of-art Facilities

The most influential PTA is the international PTA, or IPTA in short. It consists of various institutions such as European Pulsar Timing Array (EPTA), Parkes Pulsar Timing Array (PPTA), Nanohertz Observatory for Gravitational Waves (NANOGrav) and the Indian Pulsar Timing Array(IPTA) [13]. This study will mainly focus on PPTA project. Unlike the other approach which uses laser interferometers to detect GW, the PTA projects uses the data of numerous pulsars in space, requiring mainly ground-based telescopes. The Parkes project uses the Murriyang telescope to observe pulsars [14]. During its 60 years of service, it has detected almost half of the 2000 known pulsars, and could receive radio waves with wavelengths ranging from seven millimeters to four meters long.

The telescope one covered above is only one of the several parts of the Parkes project , which is consisted of telescope, receiver, online signal processing, calibration and offline signal processing. The Parkes Telescope Control System (TCS) controls the pointing of the telescope, the selection of receivers, systems and observation modes. Under its control, the telescope can be pointed with an accuracy of 11 arcseconds. The Parkes project mainly receives data on 3 radio bands, 10cm, 20cm, and 50cm, with frequency of 3100MHz, 1400MHz and 700MHz, respectively. The Parkes Multibeam receiver mainly covers data at 20cm, while the O-H receiver takes over when the Multibeam receiver is unavailable. The 10/50cm receiver deals with data on 10 cm and 50 cm bands.

Some backend systems are deployed to process the received data. The “pulsar wide band correlator: (WBC), “Parkes Digital FilterBank” systems(PDFB1-PDFB4) have similar capabilities , and are designated to compute the products of the received signals. As they are being fine-tuned overtime, the previous system will be decommissioned soon after a new system came out, for example the PDFB2 was deployed in June 2007, and in December PDFB1 was decommissioned. APSR, different from previous systems, is used as a baseband recording system. It is often used in complement with PDFB3 and PDFB4, and the recorded data will be stored in discs at Parkes. All of these backend systems are in regular communication with TCS. After online processing, the data should be calibrated for further usage. Normally the input signals will be adjusted to less than 0.5db around the operating point of 10 digitiser counts, for data acquired from PDFB systems. The PAC program from PSRCHIV is then applied to flatten the bandpass, and the PCM program from PSRCHIV utilized “measurement equation modelling” to transform the data into ‘PCM’ files. PSRCHIVE pulsar signal processing system is responsible for the off-line processing part. Off-line signal processing is important because of the uncertainties in Pulsar Times of Arrival, as well as the inconsistent between data from different instruments. The system from PSRCHIV will produce Times of Arrival using the reference template generated by the PDFB system.

## 5. Observation Results

The PPTA’s third data release mainly focuses on nano-frequency gravitational waves [15]. As a new receiver system was deployed in 2018, the “Ultra-Wide-Bandwidth Low-Frequency” (UWL) system shed new light on low frequency gravitational waves. The new UWL system has instantaneous sensitivity and is good at tackling noise processes using wide-band timing techniques. To start, the project released the data set of dozens of pulsars and their physical properties, including its period, observing time span, number of observations, and total number of Timing of Arrivals. For example, the Pulsar marked as J1600-3053 was observed using the newly available band provided by the UWL system. Observations tell that it has a period of 3.60 ms nad dispersion measure of 52.3 pc/cm$^3$. The pulsar has been observed using the UWL system for 3.71 years and 58 times, and 1303 radio bursts were received during this period. The Fig. 3 is part of the normalized intensity of the same pulsar J1600-3053 graphed with observing frequenc y(left-hand), Pulse Phase (Below) and sub-band of UWL system (right). The shaded areas indicate that this frequency band could be covered by previous receivers, like the E band roughly stands for the 1241-1497MHz or 20cm band, previously under command of the Parkes Multibeam and O-H receiver covered before. The UWL also measured normalized intensity in greater detail.

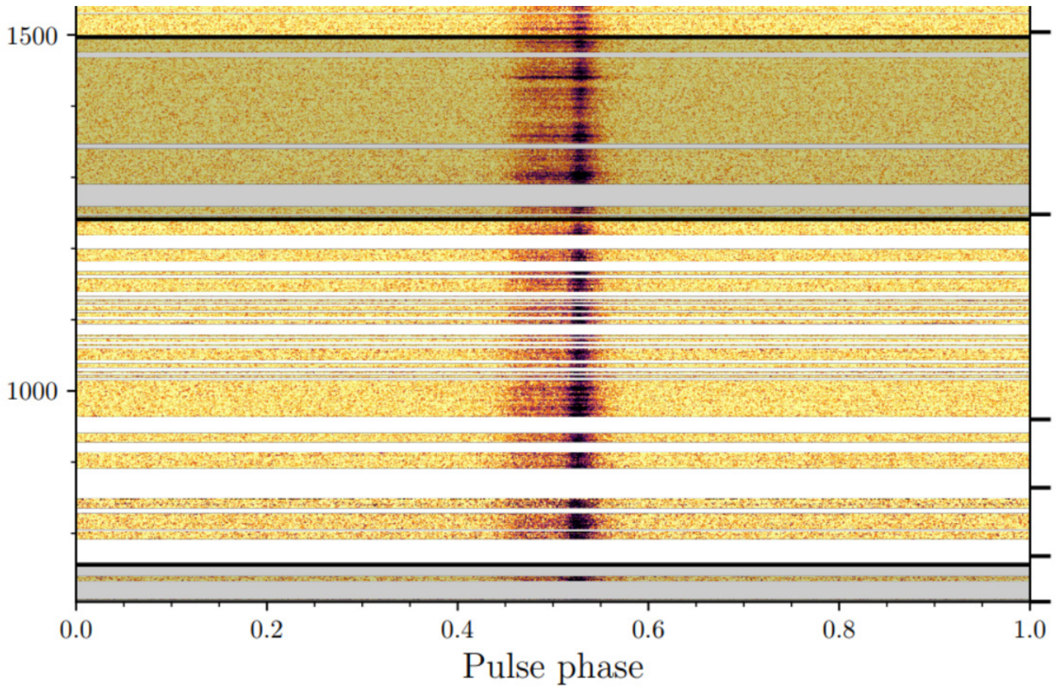

**Fig. 3 The normalized intensity of the pulsar J1600-3053 [15].**

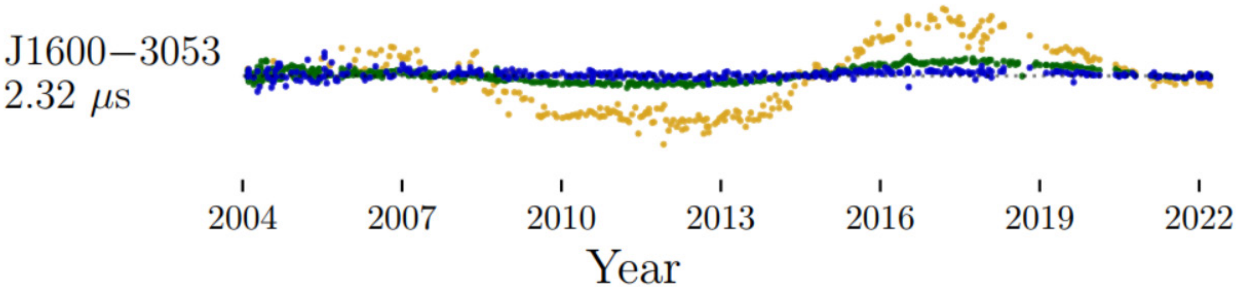

**Fig. 4 The band-averaged timing residual of the J1600-3053 [16].**

The Fig. 4 is the band-averaged timing residual of the same pulsar, J1600-3053. The different colors in the graph symbolize the difference in frequency. Blue is the 10 cm band, green is the 20cm band and gold is 40cm. The graph is constructed that the background noise is not subtracted from the residual, and the noise is also known as “stochastic gravitational wave background”. The “stochastic gravitational wave background”(GWB) pulsar timing arrays aim to describe also has nano-frequency, and is generated by the superposition of supermassive blackholes and the gravitational waves they created. Researchers believe that by looking into the stochastic GWB, just as analyzing the Cosmic Microwave Background, one could understand more about the origin and the evolution of the universe. In the second data release from the IPTA project, they determined upper and lower bounds and confidence levels for the amplitude of gravitational waves [16]. Fig. 5 the common-spectrum process parameters, or CP parameters. In the graph, the green line is the estimated value, while the yellow and blue lines are confidence intervals. By analyzing the green lines, researchers found that the expected amplitude of the gravitational wave is around $2.8\times10^{-15}$, or $A_{CP}=2.8\times10^{-15}$. The calculations and plotting involved auto +cross correlations, together with HD ORF(overlap reduction function). This result is much better than the first IPTA data release, which only set $1.7\times10^{-15}$ for the upper limit of the common-spectrum magnitude.

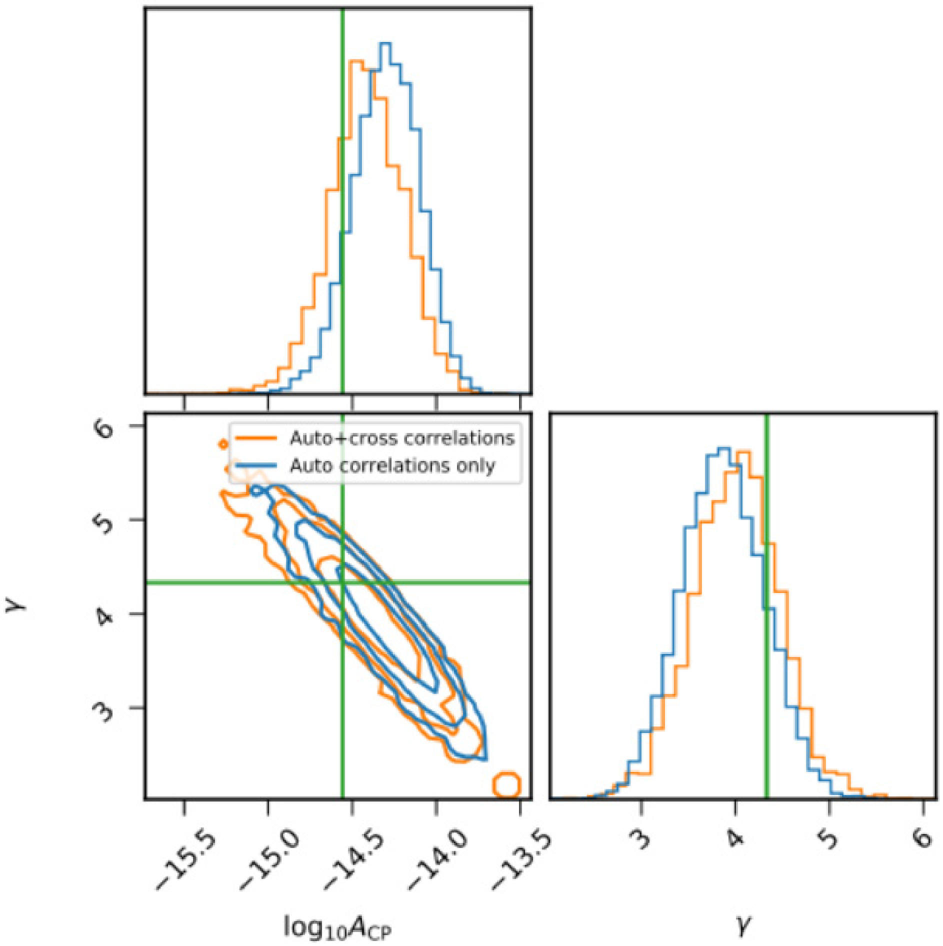

**Fig. 5 The common-spectrum process parameters [16].**

## 6. Limitations and Prospects

The PPTA and the IPTA projects have made significant achievements, but there remain some problems. Although the receiver in the Parkes program has been updated for decades, from the wide band correlator to PDFB 1 through 4, and to the latest UWL system, there are still some problems with the UWL system. Similar to Fig. 4, Fig. 6 is also the normalized intensity of the pulsar J1600-3053. The white parts in Fig 4 were covered with black in Fig. 6, which shows data loss due to technical issues such as interference. Moreover, the flux density is inconsistent in different frequency bands, leading to conjectures such as interstellar scintillation.

The IPTA project found the approximate value of the magnitude of the common spectrum, but the questions remained on its source. Researchers are convinced that these are generated by pulsars, but what about the noises from other pulsars? Moreover, the most important gravitational fields are from supermassive blackhole binaries (SMBHB) which could reveal more information about galaxy formation, and up to now their existence hasn't been detected yet. There are also some trivial problems including that the error bound for the magnitude of common spectrum is too large, or there are difficulties in accurately describing the timing of millisecond pulsars [17]. However, these problems are expected to be solved in the upcoming years, and luckily one will eventually detect more nanofrequency gravitational waves, determine the stochastic gravitational background and in this way supplement the LIGO research. The gravitational wave from various other sources might also be detected, such as primordial black holes, which is also a candidate for dark matter.

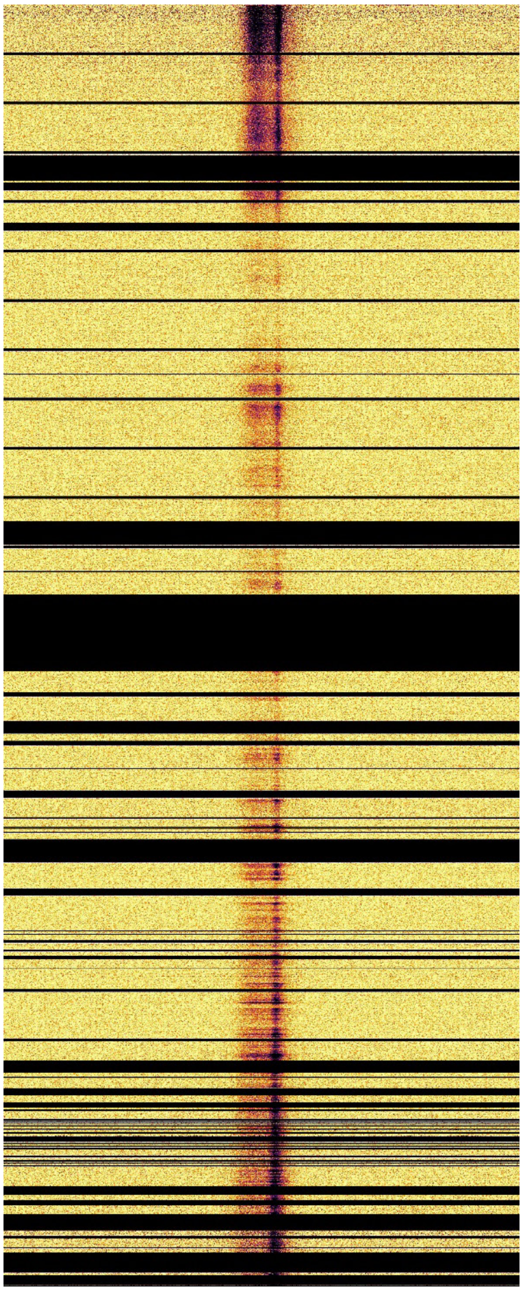

Fig. 6 The more accurate normalized intensity of the pulsar J1600-3053 [17].

## 7. Conclusion

To sum up, this study analyzes on how pulsar timing arrays help search for gravitational waves. Gravitational waves have been a tool for mankind to understand the universe, as the physical properties of the waves reveal data about celestial objects. The concept of gravitational wave aroused after some minor adjustments in the Newtonian gravitational field equation, and was discussed extensively by Einstein on its properties and applications. There have been various attempts to detect GW, with laser interferometers being the most famous and fruitful approach. However, as limitations in the arm length and the laser wavelengths, laser interferometers could not detect ultra-low frequency gravitational waves, which are expected to be detected by pulsar timing array projects. Currently several pulsar timing array projects around the world are actively recording the physical properties of pulsars and pulsar binaries. Once the stochastic gravitational wave background is determined and properties of pulsars are all recorded, the projects are anticipated to detect GW generated by “supermassive blackhole binaries”, or SMBHB. Currently the telescope, receiver, signal processing and calibration parts in a pulsar timing array are still actively being updated, so one could expect there to be more discoveries in the near future.

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