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Application of Multifractal Analysis in Stock Market

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Abstract:

Multifractal analysis provides a detailed approach to examine complex systems with varying scaling behaviors across multiple time scales. In this article, multifractal detrended fluctuation analysis (MFDFA) is applied to the index returns of Netflix from 2019 to 2024. The purpose is to uncover the multifractal nature of financial data through analyzing original, shuffled and surrogate time series and to identify sources of multifractality, particularly focusing on the roles of fat-tailed distributions and temporal correlations. This article finds out that even after shuffling, which disrupts time-dependent correlations, the multifractality still remains or even intensifies. Meanwhile, the surrogate data is investigated to study the sources of multifractality. The results show that Netflix stock returns exhibit clear multifractal properties, primarily driven by fat-tailed distributions rather than long-term correlations. In general, multifractal detrended fluctuation analysis provides important insights into the complex dynamics of financial markets, and it demonstrates how multifractal analysis can reveal underlying structures that traditional methods often overlook. These findings have implications for better risk management and market analysis by acknowledging the critical role of extreme events and distributional characteristics in stock market behavior.

Keywords: Multifractal analysis; financial time series; Netflix index returns.

1. Introduction

Multifractal analysis is an advanced mathematical framework used to study and characterize complex systems that exhibit variability across multiple scales. Unlike traditional fractal analysis, which typically assumes a unique scaling exponent, multifractal analysis accommodates a spectrum of scaling exponents, allowing for a more detailed and nuanced understanding of phenomena that are inherently heterogeneous. This makes it particularly valuable in studying real-world systems where uniform scaling laws do not apply [1].

One of the most prominent techniques within the field of time series analysis is multifractal detrended fluctuation analysis (MFDFA). It has been developed to analyze non-stationary time series data, where the mean and variance can vary over time [2]. By systematically removing trends of varying orders, multifractal detrended fluctuation analysis isolates the intrinsic fluctuations of the data, this makes the detection of multifractality easier [3]. MFDFA technique has become indispensable in various fields due to its robustness and versatility in dealing with real-world data, which are often noisy, incomplete, and influenced by external factors [4].

In finance, for instance, MFDFA is widely used to analyze financial markets, particularly in understanding price fluctuations and market volatility [5]. Huge fluctuations can influence the multifractality of time series, typically the extreme events like market crashes [6]. Traditional models that assume a random walk or a normal distribution of returns fail to capture the extreme variations observed in financial data [7]. However, MFDFA can reveal hidden multifractal structures within the data, providing deeper insights into the markets. This has significant implications for risk management and the development of trading strategies, as it allows for a more accurate assessment of market behavior under different conditions [8].

Furthermore, it is necessary to understand where does multifractality originate. The following two sources are the main causes of multifractality: distinct persistent correlations of small and large fluctuations over time, along with a broad probability density function [1, 2]. Long-range correlations involve persistent patterns or trends over different time scales, which are commonly seen in financial markets, this leads to volatility clustering and complex scaling behaviors [9]. Meanwhile, fat-tailed distributions reflect the presence of extreme events that occur more frequently than what a normal distribution would predict. These heavy tails contribute significantly to multifractality, even when the time series is randomized, which removes correlations but retains the distribution. The persistence or even increase in multifractality after shuffling highlights that fat-tailed distributions are a key driver of this phenomenon. Studies have shown that in financial data, such as stock returns, the broad distribution plays a more dominant role in causing multifractality than time-dependent correlations [10]. In this article, MFDFA is applied to analyze the index returns of Netflix and the sources of multifractality.

2. Methodology

2.1 Data Source

The dataset used in this article comes from Yahoo Finance. This data contains daily stock trading information of Netflix with currency in USD and it ranges from 20 August 2019 to 20 August 2024, which is around 5 years.

2.2 Method Introduction

According to Kantelhardt et al., there are five steps in MFDFA. Suppose x_k, where $k=1,\ldots,N$, is a time series, and N is the length of the series [1].

Step 1: Find the profile:

$$y(i)\equiv\sum_{k=1}^{i}\left(x_k-\bar{x}\right), i=1,\ldots,N \tag{1}$$

where $\bar{x}$ denotes the mean of the time series.

Step 2: The profile $y(i)$ is then partitioned into $N_s\equiv int\left(\frac{N}{s}\right)$ non-overlapping segments of equal length s. Normally, the length of the time series is not a multiple of the chosen time scale s, a small part at the end of the profile can still be considered by repeating the same procedure from the other side. Thus, there are $2N_s$ segments in total.

Step 3: Find the local trend for each of the $2N_s$ segments using least square fit. Then, the variance is given by:

$$F^2(v,s)\equiv\frac{1}{s}\sum_{i=1}^{s}\{y[(v-1)s+i]-y_v(i)\}^2 \tag{2}$$

for $v, v=1,\ldots,N_s$ and,

$$F^2(v,s)\equiv\frac{1}{s}\sum_{i=1}^{s}\{y[N-(v-N_s)s+i]-y_v(i)\}^2 \tag{3}$$

for $v=N_s+1,\ldots,2N_s$, where $y_v(i)$ denotes the fitting polynomial in segment v.

Step 4: Taking average over all segments, the qth order fluctuation function is:

$$F_q(s)=\left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}\left[F^2(s,v)\right]^{\frac{q}{2}}\right\}^{\frac{1}{q}} \tag{4}$$

The index variable q can be any non-zero real number.

Step 5: The scaling behavior of the fluctuation functions are determined by analyzing log – log plots $F_q(s)$ versus s for each value of q. If the time series x_k are long-range power-law correlated, $F_q(s)$ increases, for large values of s, as a power-law:

$$F_q(s)s^{h(q)} \tag{5}$$

For large time scales, $F_q(s)$ is statistically unreliable since there are only a few segments. When $q=2$, $h(2)$ is same as the Hurst exponent, so the function $h(q)$ is defined to be the generalized Hurst exponent. The value $h(0)$, which is identical to the limit of $h(q)$ as $q\to 0$, cannot be calculated directly from equation (4) due to the exponential singularity. Hence, the following method can be applied:

$$F_0(s)\equiv exp\left\{\frac{1}{4N_s}\sum_{v=1}^{2N_s}ln\left[F^2(v,s)\right]\right\}s^{h(0)} \tag{6}$$

In case of monofractal time series, $h(q)$ does not depend on q as the time series is fully described by a single exponent.

3. Results

3.1 Multifractal Analysis

MFDFA is applied to the logarithmic index returns of Netflix. The returns are calculated as:

$$r_t = ln(P_t) - ln(P_{t-1}), \tag{7}$$

where p_t is the closing index at time t.

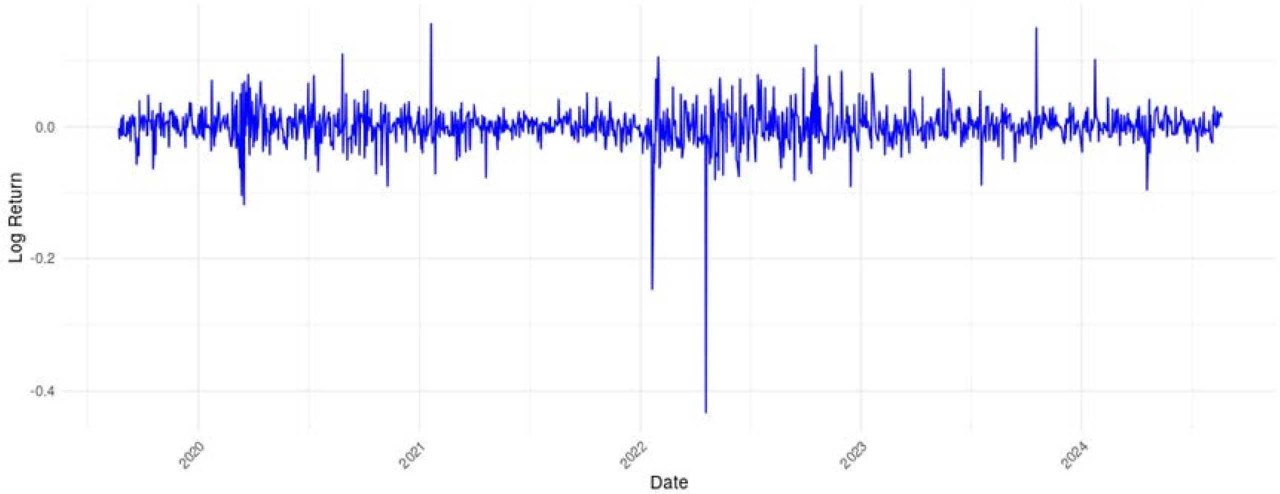

Fig. 1 The index returns of Netflix

Figure 1 shows the logarithmic index returns of Netflix; it is obvious that fluctuations persist across the entire series.

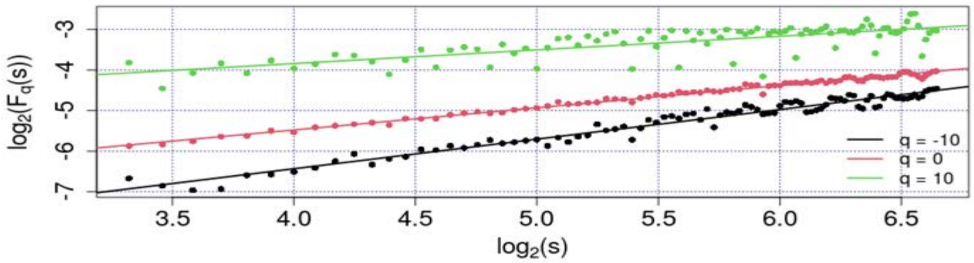

Fig. 2 Log-log plot of $F_q(s)$ versus s

Figure 2 presents a log-log plot of the fluctuation functions $F_q(s)$ against the time scale s for three different values of q. The distinct slopes for different q values indicate the multifractal characteristics of the series. It is also clear from the plot that as q varies, the scaling behavior changes, which is a clear sign of multifractality. For $q = 10$, the curve is relatively flat, indicating that larger fluctuations dominate at this moment order. Conversely, for $q = -10$, the steeper slope suggests that smaller fluctuations are more relevant at this moment order. The middle curve at $q = 0$ represents the behavior around the median fluctuations. The divergence of these curves with increasing s demonstrates that the scaling properties differ across different magnitudes of fluctuations, underscoring the multifractal nature of the Netflix returns.

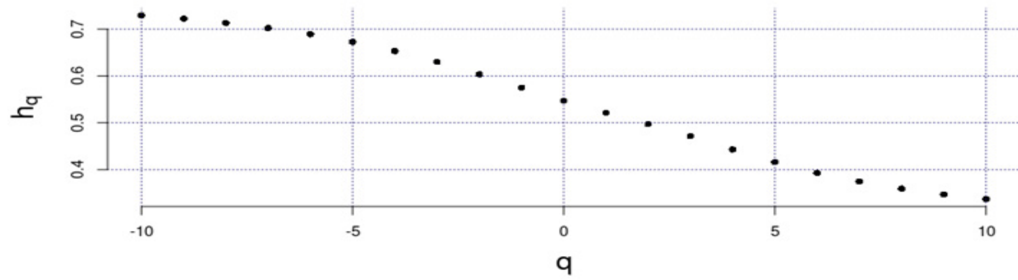

Fig. 3 Hurst exponent of Netflix returns

In Figure 3, the Hurst exponent h_q is plotted against the moment q. the Hurst exponent h_q decreases as q increases, indicating the presence of multifractality. This trend suggests that fluctuations of different magnitudes exhibit distinct scaling behaviors. Specifically, for large negative q, h_q primarily captures the small fluctuations in the time series, while large positive q captures larger fluctuations. The decreasing nature of h_q as q increases suggests that large fluctuations are less persistent compared to small ones. This behavior is typical in financial data, where small fluctuations tend to be more frequent, while large fluctuations are rarer and exhibit different dynamics.

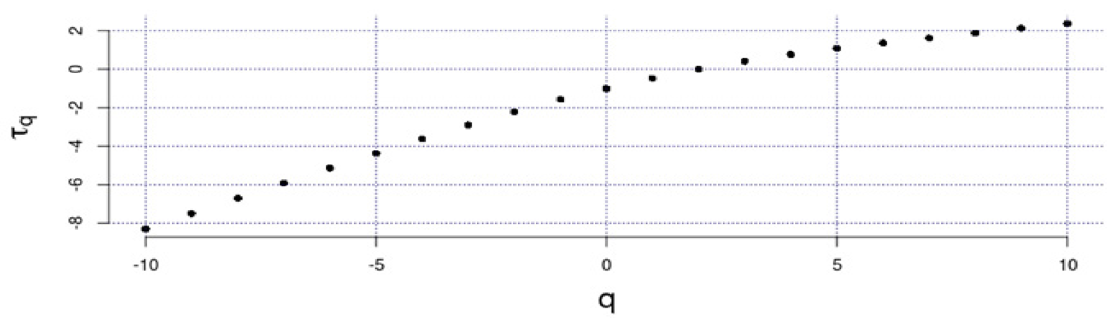

Fig. 4 Mass exponent τ_q versus q

As shown in Figure 4, the nonlinear curve in this plot indicates that the scaling behavior is not uniform across different q values. In a monofractal process, τ_q would exhibit a linear relationship with q. However, the convex shape of this curve suggests that the Netflix return data exhibits multifractality. The slope of τ_q is steeper for negative q values, which again emphasizes the significance of smaller fluctuations. For positive q, the curve flattens, reflecting that large fluctuations contribute less to the overall scaling behavior. This asymmetry in the curve is a hallmark of financial time series, where small and large price changes follow different scaling laws due to varying market dynamics.

Figure 5 shows the multifractal spectrum, where α and $f(\alpha)$ denote the singularity strength and the fractal dimension respectively. The bell-shaped curve observed in the plot is characteristic of a multifractal spectrum. The shape of the curve tells the strength of the multifractality, it means a broader multifractal spectrum indicates a greater level of multifractality. In this case, the curve is relatively wide, suggesting that the Netflix return series exhibits strong multifractality. The leftward tail of the spectrum, corresponding to lower α values, which indicates the presence of singularities associated with small fluctuations. On the right side, the spectrum extends to higher α values, indicating singularities related to larger fluctuations. The asymmetry of the curve, which leans more toward higher α, suggests that larger fluctuations are more diverse and play a significant role in the series' behavior.

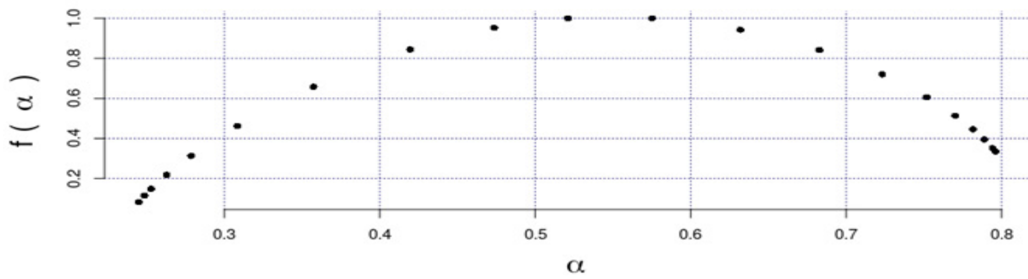

Fig. 5 Multifractal spectrum of Netflix

3.2 Sources of Multifractality

Understanding the nature of multifractality is crucial. Typically, the two main sources of multifractality in time series are fluctuations in long-range temporal correlations and heavy tail probability distributions. Through the analysis of shuffled and surrogate time series, these sources can be identified. Since shuffling the series eliminates long-term correlations, so that fat-tailed distributions become the only source, while the surrogate data preserves correlations but normalizes the distribution. If long-range correlations are the sole cause, the shuffled data will approximate a constant value near 0.5 for all q, and the multifractal spectrum typically becomes narrower. However, when the multifractality is driven solely by distribution's heavy tails, the surrogate data will show a significant reduction in the multifractality, meaning that heavy-tailed distribution was the dominant cause. The multifractality of the shuffled series will reduce, if the multifractality comes from both sources. The degree of multifractality can be characterized by:

$$?\alpha = \alpha_{max} - \alpha_{min} \tag{8}$$

Table 1. Multifractal degrees of Netflix's return

	α_{max}	α_{min}	$?\alpha$
Original series	0.7962	0.2450	0.5512
Shuffled series	0.8224	-0.0853	0.9077
Surrogate series	0.7103	0.2662	0.4441

It is apparent from Table 1 that shuffling the series made the multifractality stronger. In financial time series, an increase in multifractality is often observed after shuffling, which occurs when all temporal correlations are eliminated but the distribution is maintained. This suggests that the broad distribution plays a significant role in driving the multifractality, even more than time correlations [9]. Additionally, there is a reduction in the multifractality of the surrogate data, this confirms that heavy-tailed distribution was the main source. The dominance of fat-tailed distributions can also be seen by analyzing the statistical data of the time series.

Table 2. Statistical analysis of the Netflix returns series

Mean	Median	Maximum	Minimum	SD	Skewness	Kurtosis	Observations
0.0007	0.0008	0.1558	-0.4326	0.02957	-2.6511	44.7412	1258

Table 2 gives an overview of the statistical features of the Netflix returns. There is a skewness of -2.6511 and a kurtosis of 44.7412, this indicates heavy tails and a peaked distribution. Therefore, the data contains many outliers compared to a normal distribution, confirming the dominance of heavy-tailed distribution.

4. Conclusion

The analysis of multifractality in financial markets is essential for understanding the complex and diverse scaling behaviors that influence market dynamics. This article studied the index returns of Netflix from 2019 to 2024 by using MFDFA to discover the multifractality of the data and its underlying sources. The findings indicate that the Netflix time series exhibits significant multifractality, primarily driven by fat-tailed distributions rather than persistent temporal correlations.

In financial time series such as Netflix's returns, multifractality can originate from two primary sources: persistent temporal correlations and the underlying statistical distribution of the data. It has traditionally been assumed that temporal correlations are the primary factor driving multifractality in time series analysis, as they significantly influence the scaling properties and complexity of the data across different time scales. However, the results of this study challenge that assumption by highlighting the significant role of fat-tailed distributions. The analysis showed that even after shuffling the time series, destroying any time correlations, the multifractality either persists or intensifies. This suggests that the broad, fat-tailed nature of Netflix's returns is a more substantial contributor to multifractality than previously thought.

This finding has important implications for financial modeling and risk management. Traditional financial models often rely on assumptions of normally distributed returns or simple random walk behavior, which fail to capture the true complexity of market dynamics. By learning the impact of fat-tailed distributions, models can be adjusted to better predict market behavior, especially in extreme scenarios. For Netflix's index returns, recognizing the role of fat tails in multifractality can lead to more accurate assessments of market risks and the development of more effective trading strategies. Furthermore, the study highlights that the multifractal spectrum of Netflix's returns broadens after shuffling, indicating stronger multifractality, and the wider spectrum reflects greater heterogeneity in scaling behaviors. This observation underscores the need for financial models that can accommodate the inherent complexities of real-world data, moving beyond simplistic assumptions.

In conclusion, the analysis of Netflix's index returns shows strong multifractality and provides evidence that heavy tail distributions are a key driver of multifractality in financial data. The persistence and increment of multifractality after shuffling confirm that distributional characteristics play a critical role, even more than the influence of temporal correlations. These findings offer valuable insights into market behavior and emphasize the importance of considering fat-tailed distributions in financial analysis. As markets continue to evolve, embracing models that account for both temporal correlations and distributional properties will be crucial in navigating the complexities of financial systems.

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