

Analysis of the Principle of Gravitational Wave Detection and State-of-art Results

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Abstract:

Over one hundred years after Einstein's first theoretical prediction of gravitational waves, the recent observations of gravitational waves by interferometric detectors have risen excitement in the science community with the light from a new era of directly observing perturbations in spacetime itself. With the large quantity of new research in the gravitational wave detection area, this study provides a brief summary of the current achievements of the field. This paper first gives a short introduction of the field, then offers a summarize of the theories and principles of gravitational wave detection. Afterwards, it demonstrates modern detectors using the advanced LIGO detectors as an example, and presents its groundbreaking results, signal GW150914 and GW151226. Finally, the limitations and prospects of current observations are proposed. By summarizing fundamental concepts and major achievements in the field of gravitational wave detection, it is hoped to provide future students and researchers with a basic idea of the field and thus promote new ideas and advancements in the rising era of gravitational astronomy.

Keywords: Gravitational wave detection; interferometric detectors; LIGO.

1. Introduction

The first theoretical prediction of gravitational waves (GWs) was given by Einstein in 1916 [1], then later corrected by himself in 1918 [2]. Einstein claimed that relativistic gravity has wave solutions, which pictures transverse waves of spatial strain that are generated by accelerating masses and travel through space in the speed of light [1, 2]. It was understood since Einstein that GWs, if they really exist, should have remarkably tiny amplitudes. However, after proofs of general relativity such as the observation of gravitational lenses turned out to be successful, observation of GWs was seen as a new test to general relativity [3]. The first indirect proof of the existence of GWs was from the discovery of the binary pulsar system PSR B1913+16, in which the mass and acceleration of both objects was large enough for energy loss from GWs to cause observable effects during the observation period [4]. After this major breakthrough, the subject was soon turned to detecting GWs directly for quantitative tests of general relativity as well as a new point of view for astronomical observations. The first device to be proposed was the resonant mass detectors in the 1960s, but in a global network of them they failed to detect GWs [5]. Interferometric detectors, on the other hand, were first proposed in the early 1960s and 1970s, after which stud-

ies of the concepts, performances and noise led to the idea of long-baseline broadband laser interferometers that are potentially capable of significant improvements in sensitivity [3]. In the 2000s various detectors of the kind were built, including the Laser Interferometer Gravitational-Wave Observatory (LIGO) in the United States, GEO 600 in Germany and Virgo in Italy [3]. Early observations by these detectors together set upper limits to GW observations, as well as advancing their instrument sensitivity and gradually forming a global network [3]. The first GW signal to be observed was GW150914, detected on September 14, 2015 at 09:50:45 UTC by the advanced LIGO detectors [3]. At the time of the detection, the advanced LIGO detectors had already went through a major upgrade, but had not yet reached their designed sensitivity [6]. Soon after its first discovery, another signal, GW151226, was observed by the advanced LIGO detectors, on December 26, 2015 at 03:38:53.647 UTC [7]. Both times the data was in consistence with predictions from general relativity, providing new, important proofs in the high-velocity, strong-field regime [6, 7]. After the initial success of the advanced LIGO detectors, other detectors around the world have also joint the international network of interferometric GW detectors, such as Virgo in Italy and GEO600 in Germany. With more detectors in the global network, precise localization of the source in the

sky becomes possible, joining GW detection with other methods of observation and opening new eras of astrophysics.

With all the exciting recent discoveries, this paper aims to provide a brief analysis on the principles of interferometric GW detection and the state-of-art results, as well as provide insights to improvements in future observations. In Sec. 2, a brief description of GWs according to the theory of general relativity will be offered. Sec. 3 will introduce the basic components and principles of interferometric GW detectors, while Sec. 4 will demonstrate how modern-day advanced interferometric detectors are designed to minimize the level of noise and the sources of remaining noise, both using the advanced LIGO detectors as an example. In Sec. 5, the two earliest results were presented and analyzed. As for Sec. 6, current limitations of GW detection are assessed, together with advice on future improvements and prospections. Finally, a summary of the paper will be found in section 7.

2. Descriptions of GW

GWs are ripples of curvature that propagate through spacetime itself at the speed of light [8]. Similar to electromagnetic waves which are caused by acceleration of charges, GWs are caused by the acceleration of masses in spacetime[8]. Yet unlike electromagnetic waves, GWs usually have a quadrupole structure and wavelengths which are longer than the size of the source [8]. Since GWs are changes in the spacetime itself and do not interact strongly with matter, the current method of detection aims to detect the changes of amplitude with time by measuring the change of strain of spacetime with time [8]. Here, a basic idea of GWs in general relativity will be provided. For a more detailed introduction [9]. According to the modern benchmark model of cosmology, the universe is generally flat. In this case, an easy approximation would be linearized gravity, where the spacetime metric, g_{ab} , is extremely similar to the flat metric, η_{ab} , the metric which is used in special relativity:

$$g_{ab} = \eta_{ab} + h_{ab}, |h_{ab}| \ll 1. \quad (1)$$

In such linearized situations, the Einstein equation would be:

$$\square \bar{h}_{ab} = -16\pi T_{ab}. \quad (2)$$

which allows a class of solutions for plane waves propagating in spacetime:

$$\bar{h}_{ab}(x, t) = \text{Re} \int d^3k A_{ab}(k) e^{i(k \cdot x - \omega t)}. \quad (3)$$

For the sake of specialization, a further restriction of global vacuum spacetime in which $T_{ab} = 0$ everywhere and is gradually becoming flat ($h_{ab} \rightarrow 0$ as $r \rightarrow \infty$) is assumed. In

this case a transverse-traceless gauge(TT-gauge) can be applied, with the metric perturbation in such gauge written as h_{ab}^{TT} . The TT-gauge has a wave solution, but more realistic solutions do not always apply to such restricted preconditions. Yet it is provable that only the TT-gauge parts have a real wave solution and transfer energy, making it the only parts to consider in actually situations.

3. Principle of Detection

Since the first theoretical hypothesis of GWs by Einstein, it was soon realized that because of the weakness of the gravitational force, the strain amplitude of GWs would be extremely small [8]. To detect the tiny strains, an instrument that is be capable of measuring almost negligible changes of length over enormous distances is required. A Michelson interferometer is a one of the instruments which is capable of detecting those very small changes in length over relatively long distances in the case of relatively high frequencies [9]. Many modern-day GW detection projects, including LIGO, Virgo, GEO600, TAMA300 and ACIGA, focuses on detectors that are based on the Michelson interferometer [9].

A Michelson-based interferometer usually consists of a laser source, a bean-splitter, two perpendicular based arms and a photodetector [3]. When the interferometer is at work, the laser acts as a strong monochromatic light source, with the light produced by which split by the bean-splitter into the two perpendicular arms [3]. The light in each arm travels towards the end of either arm and is reflected back to the bean-splitter by the two mirrors placed at both ends [3]. There, the light is directed to the photodetector, where interferometric fringes can be observed because of optical path difference(OPD) of the two routes of light [3]. Because of its special mechanism, any change in the OPD which is over the length of one wavelength of the light produced by the laser will cause a significant change in the interferometric fringes. This property makes the Michelson interferometer incredibly sensitive to changes in its two arms.

In the case of using a Michelson-based interferometer for GW detection, this extraordinary sensitivity is used to detect the changes in length of the two arms caused by a GW passing through [3]. To ensure that the signal detected is not a noise signal, multiple detectors which work by the same mechanism are placed apart [3]. Only coincident signals that appear in at least two detectors with the correct time difference can be suspected as GW candidates [3]. This lowers the effect of noise on the detection of GWs.

4. Detection Facilities

The following part will describe Michelson interferometer

based GW detectors using the advanced LIGO detectors as an example. The advanced LIGO detectors are state-of-art GW detectors that have made the world's first discovery of GW signals [3]. The advanced LIGO detector is built for achieving as high accuracy as possible. To reach this, the detector has to have an optimum antenna length, which, similar to electromagnetic wave receivers, is a quarter wavelength [6]. For a GW with the frequency of about 100Hz, which resembles a typical GW signal that the detector would observe, is about 750km [6]. Building such an antenna is, of course, not a very sensible choice. Instead, the advanced LIGO detectors have arms that are each 4 km long and uses a pair of test masses in each arm, which each form optical resonators that multiplies the effective length of the arms by the effective number of trips back and forth taken by light in the arms [6]. This technique greatly reduces the physical size of the detector, but has the potential of enlarging the background noise signals since the noise caused by the displacement of the test masses are multiplied by the same factor [6].

During detection runs, the advanced LIGO detectors face varied sources of noise signals. The main known noise sources are quantum noise, thermal noise, seismic noise, Newtonian noise as well as other degrees of freedom of the detector [6]. To reduce the effect of noise on GW detection and increase the signal-to-noise ratio (SNR), the following are considered when designing the advanced LIGO detectors. The main source of quantum noise arises from the Poisson-distributed arrival rate of photons, and mainly comes as radiation pressure noise at low frequencies and photon shot noise at high frequencies. The former is caused by the momentum transfer of individual photons hitting the test mass. The latter is caused by statistic fluctuations of the photon arrival times at the output of the interferometer, which is a fundamental limit in the detector's sensitivity. It can be reduced by increasing the power of the laser which is circulating in the interferometer's arms. The advanced LIGO detectors used 100kw of circulating laser power in its first observation run and plans to increase the circulating laser power in its arms up to about 750kw in the following years in to reduce the effect of photon shot noise [6].

Thermal noise are caused by thermal driven motion, which mainly arises from the Brownian motion of the test masses and their suspension systems, as well as the mechanical loss of mirror optical coatings. To reduce the impact of thermal displacement noise, the mirror coating uses a dielectric multilayer of silica and titania-doped tantala [10]. Seismic noise arises from earth-based distributions such as tidal motion and microseismical activity. To reduce the effect of seismic noise, the test masses are hung by multistage pendulum systems which are placed

on top of actively controlled seismic isolation platforms. The multistage pendulum system consists of 4 stages and has a resonance frequency ranging from 0.4 to 13 Hz, so provides excellent 7 orders of magnitude protection for the main detection wavelengths of the detector, yet limiting the effective detection frequency range to 10Hz and above. The actively controlled seismic isolation platforms then provides an extra 3 orders of magnitude attenuation, resulting in a total protection of 10 orders of magnitude. Newtonian noise arises from perturbations in the local gravitational field which are caused by changes in mass distributions. This form of noise does not have a great impact on the detector's sensitivity currently, but might become a limiting factor for future observations of lower frequencies and thus require active cancelling [6].

There are still various other noise sources, both identified and unknown, all summarized into the term other degrees of freedom. To minimize the effect of these noise sources on GW detection, each of the advanced LIGO detectors are equipped with seismometers, accelerometers, microphones, magnetometers, radio receivers, weather sensors, ac-power line monitors, and a cosmic ray detector [6]. Besides, only coincident detector between the two advanced LIGO detectors can be considered as GW candidates [3, 6].

5. Observation Results

The first known observation result of GW signals is the GW150914 signal observed on September 14, 2015 at 09:50:45 UTC by the two advanced LIGO detectors, located in Hanford, Washington (H1) and Livingston, Louisiana(L1). GW150914 appeared in both detectors with a 10ms time difference, which is the time required for a GW to travel from one of the two advanced LIGO detectors to the other. After combining the two coincident events detected in both detectors, the signal was shown to have a very high SNR of 24 [3]. The details of GW150914 is shown in Fig. 1. The frequency of GW150914 starts at 35Hz and levels up to 250Hz, with a peak gravitational strain 1.0×10^{-21} [3]. By combining GW150914 to numerical solutions of binary black hole collision from general relativity, it can be shown that the data fits well with predictions based on the theory of general relativity with the mass of the two initial black holes estimated to be $36^{+5}_{-4} M_{\odot}$ and $29^{+4}_{-4} M_{\odot}$, and a final black hole mass of $62^{+4}_{-4} M_{\odot}$, which means that $3.0^{+0.5}_{-0.5} M_{\odot} c^2$ of energy was been radiated in the form of GWs. It is estimated that the chance of the signal being a false alarm is smaller than 1 event per 203000 years, making it generally credible[3]. Moreover, GW150914 is also the first observed binary black hole merging event and confirms the existence of

such binary black hole systems [3]. On December 26, 2015 at 03:38:53.647 UTC, quite soon after the observation of GW150914, another GW signal was detected by the advanced LIGO detectors, which was named GW151226 [7]. Because of the small strain amplitude, longer lasting time (approximately 1s) and wide frequency range from 35 to 450Hz with about 55 cycles, the signal was detected by an online matched-filter search, instead of the generic transient searches which identified GW151226 [7]. GW151226 has a SNR of 13, relatively small compared to that of GW150914, yet still having a probability of being a false alarm smaller than 1 event per 1000 years according to the online match-filter searches, so can be confidently identified as a true GW signal. Further analysis of GW151226 by comparing the data with numerical predictions given by general relativity suggests that GW151226 is most probably caused by two black holes of a binary black hole system colliding into each other, with initial black hole masses of $14.2^{+8.3}_{-3.7}M_{\odot}$ and

$7.5^{+2.3}_{-2.3}M_{\odot}$ (source-frame), and the black hole left after the collision has a source-frame mass of $20.8^{+6.1}_{-1.7}M_{\odot}$ [7]. However, there is also a 4% chance for the source-frame mass of the secondary black hole to lie in the 3–5 M_{\odot} gap between the mass of known neutron stars and black holes. Due to there were only two GW detectors in observing status during the observation of GW151226 and they are the two advanced LIGO detectors which were placed in the same plain, the advanced LIGO team could not accurately confirm the source of the signal in the sky [7]. Two GW signals observed in such short time interval provides evidence that similar black hole coalescence events could be occurring in the local parts of the universe with way higher frequencies than previously expected [7]. The detection of GWs itself also declares the opening of the exciting era of GW astronomy [3, 7].

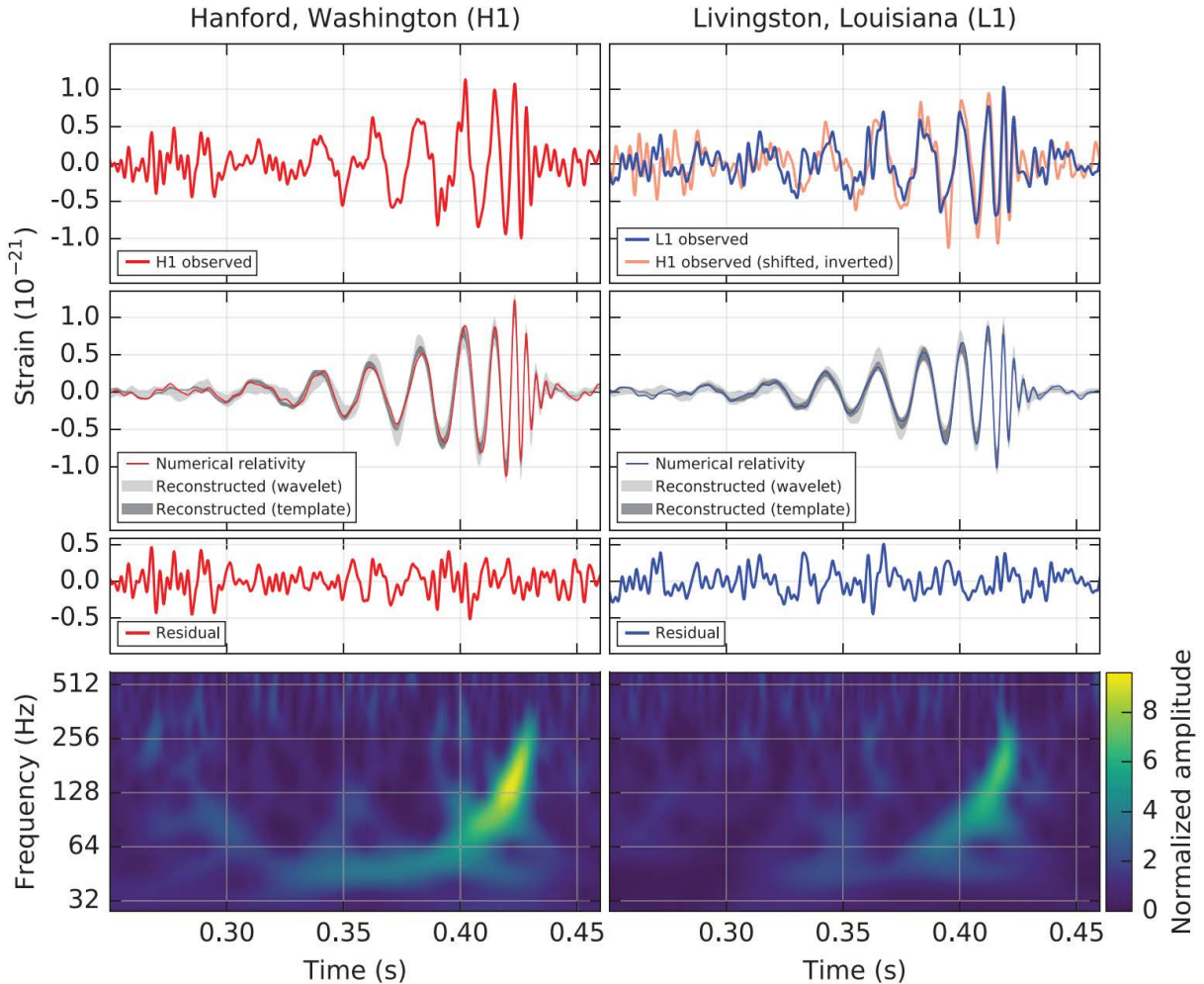


Fig. 1 signal GW150914 received by the advanced LIGO detectors [3].

6. Limitations and Prospects

The main limitation of currently operating GW detectors is the amount of background noise presented. Ground-based interferometric detectors, such as the advanced LIGO detectors shown before, are greatly effected by ground based noise sources, with majorly seismic and Newtonian noise rising significantly at lower frequencies [6]. This limits ground-based detectors to the high frequency band ($1 \lesssim f \lesssim 104$ Hz) [9]. Though the band is useful at observing events caused by large, relativistic objects, such as neutron stars and black holes, which the advanced LIGO detectors target [3, 6, 7, 9]. Yet in this waveband there are still improvements than can be done to potentially increase the sensitivity of the detector and its signal-to-noise ratio for events [6].

For the low frequency band, $10^{-5} \lesssim f \lesssim 1$ Hz, spaced-based interferometric detectors such as LISA have been proposed, which are thought to include magnificent arm-lengths of $L=5 \times 10^6$ km and orbit around the sun. A GW detector like this would be able to measure events caused by white dwarf coalescence and observe emission events which include black holes from large redshifts with great accuracy [6]. Sadly, this is now still a wonderful blueprint and have not yet put into use. For frequencies lower than this, the current-best method is to measure the arrival time pulses of distant millisecond pulsars for signs of oscillations caused by GWs. Details of relevant methods can be found [11]. Inflation, if true, will lead to a cosmic GW background in such frequency bands, so observations in this band could provide key proof to the inflation theory [6].

7. Conclusion

To sum up, this study offers a brief analysis of GW detection, including the principles and the state-of-art results. It introduces the basic ideas of GWs in general relativity and methods of detection. The leading interferometric detectors, the advanced LIGO detectors, is then used as an example to describe modern day state-of-art GW detectors as well as their noise sources and noise control precautions. After that examples of successful observations of GWs with descriptions of their data and noise analyze are given, together with limitations of current observations and expectations of the development of the field in near-future. Nowadays, with more and more advanced interferometric

GW detectors joining the global observational network and new technologies (e.g., machine-learning, aiding signal analyzing and noise cancelling), the accuracy in GW detection, including amplitude, frequency and direction, GWs are becoming more and more important sources of astronomical observations. With GWs joining electromagnetic means of observation, a new era of is opening to the astrophysics community. In this case, this paper hopes to provide an easy introduction to the recent breakthroughs in field of GW observation to aid future students and researchers.

References

- [1] Einstein A. Approximative integration of the field equations of gravitation. *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)*, 1916, 1916(688-696): 1.
- [2] Einstein A. Über gravitationswellen. *Sitzungsber. Preuss. Akad. Wiss. Berlin*, 1918, 154: 1918.
- [3] Abbott B P, Abbott R, Abbott T D, et al. Observation of gravitational waves from a binary black hole merger. *Physical review letters*, 2016, 116(6): 061102.
- [4] Hulse R A, Taylor J H. Discovery of a pulsar in a binary system. *Astrophysical Journal*, 1975, 195: L51-L53.
- [5] Astone P, Baggio L, Bassan M, et al. IGEC2: A 17-month search for gravitational wave bursts in 2005–2007. *Physical Review D—Particles, Fields, Gravitation, and Cosmology*, 2010, 82(2): 022003.
- [6] Abbott B P, Abbott R, Abbott T D, et al. GW150914: The Advanced LIGO detectors in the era of first discoveries. *Physical review letters*, 2016, 116(13): 131103.
- [7] Abbott B P, Abbott R, Abbott T D, et al. GW151226: observation of gravitational waves from a 22-solar-mass binary black hole coalescence. *Physical review letters*, 2016, 116(24): 241103.
- [8] Le Tiec A, Novak J. Theory of gravitational waves. *An Overview of Gravitational Waves: Theory, Sources and Detection*. 2017: 1-41.
- [9] Flanagan E E, Hughes S A. The basics of gravitational wave theory. *New Journal of Physics*, 2005, 7(1): 204.
- [10] Granata M, Saracco E, Morgado N, et al. Mechanical loss in state-of-the-art amorphous optical coatings. *Physical Review D*, 2016, 93(1): 012007.
- [11] Detweiler S. Pulsar timing measurements and the search for gravitational waves. *Astrophysical Journal*, 1979, 234: 1100-1104.