# **Diet problem by using Linear Programming**

Shengtai Ding, Ruokai Wang, ke Hu

#### Abstract

This paper will discuss the eating problems people encounter daily, mainly using Stigler's idea. This dietary problem refers to how people can buy the food they need at the most affordable cost under balanced nutrition. Therefore, this paper uses linear programming to optimize this dietary problem. It contains nine nutrients, each of which has a minimum value. However, two of them, vitamin A and calcium, were set to have the highest intake amount. These intake settings are universal to all human beings, without exception. The algorithm applied to solve this linear programming is a simplex method, and a positive result is obtained. Our resulting minimum cost is even slightly cheaper than Stigler's result. We conclude that if the taste of foods is not considered, people can obtain the required nutritional content by spending very little money. While it might not seem useful in modern countries, it is a meaningful investigation for underprivileged people who live in extreme poverty. However, the number of food considered in this paper is still limited. Also, the data we included come from the twentieth century for comparison with Stigler, which is already out-of-date. Therefore, given that the diet problem is a topic worth investigating, further research with more comprehensive food choices and more up-to-date data is needed.

Keywords: Optimization, minimize, standard form, canonical form, constraints

# 1. Introduction

A healthy and balanced diet is essential to improve your quality of life. As stated by Healthline, A Balanced diet "gives your body the nutrients it needs to function correctly. To get the nutrition you need, most of your daily calories should come from". According to the World Health Organization, "A healthy diet is essential for good health and nutrition. It protects you against many chronic non communicate diseases, such as heart disease, diabetes and cancer". In a typical dietary problem, the cost function is minimal while the nutritional requirements are met [1]. As everybody knows, in the early time, George J. Stigler suggested the first diet problem model mainly based on economy and simplex algorithm, and Dantzig did the same thing on the first test problem [2]. Obviously, this paper propose a diet problem by using linear programming.

Programming problems are concerned with the efficient use or allocation of limited resources to meet desired objectives. For instance, the manufacturing company that must determine what combination of available resources will make it manufacture products in a way which not only satisfies its production schedule, but also maximizes its profit [3]. But, a very special subclass of programming problems called linear-programming problems, which differs from the ordinary one in that the relationships of model can be called "linear". Spawning it off, the simplex method is the one thing that is important for solving linear program. This idea of method is firstly introduced by Dantzig, which is vital basis for this paper. Nutritional needs and balance are all the more important in our lives. But, actually, how to use the most feasible way to allocate their own money to buy the most suitable food to achieve this nutritional balance is a matter of the human life. In the article cost of subsistence, there are nine essential nutrients[4]. They respectively are calories, protein, calcium, iron, Vitamin A, Thiamine, Riboflavin, Niacin, and Ascorbic Acid, and the minimum allowance of each one is certain. Moreover, the highest intake amount of calcium and vitamin-A is added to our linear solution. The reason for limitation of calcium is that too much calcium can interfere with the functions of the nervous and muscular systems and decrease the body's absorption of zinc and iron[5]. For limitation of vitamin-A, excessive intake of it can lead to a variety of health issues, including nausea, vomiting, anorexia, headaches, blurry vision, hair loss, muscle weak- ness, and altered mentation[6]. Thus, the highest limitations of these two nutrients are worth to consider in this programming. Simply talking, people always want to be as cheap as possible, and with price rises with the corona virus still

continuing, we want to find the most affordable way to give us the nutrients we need. This paper is for us to minimize the cost of diet with the optimal amount of nutrient for human beings.

# 2. Assumption

2.1 People of all ages are in good health, need same balanced nutrition, have no fatal reactions to various food intakes, and are not

#### picky eaters.

Some diseases: like diabetes, so we have to assume that all these are not considered.

# 2.2 Prices between all food are independent, and are unchanged in the long-run.

Some economic phenomenon in real life also needs to be excluded in or modelling, about some demand and supply principle

2.3 The content of each nutrient in various foods remains unchanged in the raw and cooked state, and does not be affected by external ingredient.

Such as condiments, changeable weather and different seasons.

# 3. Formulation

Our project's target is to minimize the cost of diet of a normal American weighting 140 pounds, while fulfilling the basic nutrient requirement. In addition to the lowest requirements of nine most common nutrients, the amount of calcium and vitamin A should not exceed a certain value, or otherwise it would be detrimental to human beings.

The following is our formulation:

Minimize  $c^T x$ 

Subject to.  $Ax \ge b_L$ , A(mask; :)  $x \le b_u$ ,  $x \ge 0$ A: the amount of 9 nutrition in all food included; x: the particular cost on buying each food to reach minimum overall cost;

b<sub>L</sub>: the standard for the lowest nutritional requirements;

b<sub>u</sub>: the highest nutritional limitations of calcium and vitamin-A;

A(mask,:): the amount of calcium and Vitamin A in all food;

 $c^{T}$ : the unit-price of every commodity (Here the unit-price is all \$1)

# 4. Methodology

## 4.1. Preparation work

Before we start, we have obtained the data of food price, food nutrition content and human nutrition requirement from George J. Stigler's paper published in 1945: '*The Cost of Subsistence'*[4]. The highest limitation of calcium and vitamin-A is gained from the internet.

#### 4.2 Obtaining the standard form

#### $Min.c^{T}x$

### S.t.Ax $\geq b, x \geq 0$

We include 77 food types and 9 main nutrients. So, the matrix we obtain directly from the data table is 77 by 9[4]. 9: the number of nutrients 77: the choices of food

In order to consider the upper bound, we add two more columns to the matrix. These two columns consist the negative of nutrition contents of vitamin A and calcium in all foods. We transpose the matrix and get our A, which is 11 by 77.

2.2	40	1.1	18	11.1	3.6	1.3	10	1998	-11.1	-1.1
2.4	138	3.7	80	69	4.3	5.8	37	862	-69	-3.7
2.6	125	4	36	7.2	9	4.5	26	5369	-7.2	$^{-4}$
2.7	73	2.8	43	188.5	6.1	4.3	89	608	-188.5	-2.8
0.9	51	3	29	0.9	1.4	1.4	9	313	-0.9	-3
0.4	27	1.1	22	112.4	1.8	3.4	11	449	-112.4	-1.1
5.8	166	3.8	59	16.6	4.7	5.9	21	1184	-16.6	-3.8
14.3	336	1.8	118	8.7	29.4	7.1	198	2522	-8.7	-1.8
1.1	106	0	138	918.4	5.7	13.8	33	2755	-918.4	0
9.8	138	2.7	54	890.7	8.4	5.4	83	1912	-890.7	-2.7
9.7	20	0.4	10	21.5	0.5	1	31	196	-21.5	-0.4
3	8	0.3	8	0.8	0.8	0.8	5	81	-0.8	-0.3
2.4	16	0.4	8	2	2.8	0.8	7	399	-2	-0.4
0.4	83	0.3	12	16.3	1.4	2.1	17	272	-16.3	-0.3
1	54	2	65	53.9	1.6	4.3	32	431	-53.9	-2
7.5	864	4	134	3.5	8.3	7.7	56	0	-3.5	$^{-4}$
5.2	136	0.2	16	12	1.6	2.7	42	218	-12	-0.2
2.3	138	0.6	45	84.9	4.9	2.5	37	870	-84.9	-0.6
1.3	63	0.7	38	53.2	3.4	2.5	36	1253	-53.2	-0.7
1.6	71	0.6	43	57.9	3.5	2.4	67	862	-57.9	-0.6
8.5	87	1.7	173	86.8	1.2	4.3	55	57	-86.8	-1.7
12.8	99	2.5	154	85.7	3.9	4.3	65	257	-85.7	-2.5
13.5	1041	2.5	136	4.5	6.3	1.4	24	136	-4.5	-2.5
20	1367	4.2	845	2.9	28.7	18.4	162	0	-2.9	-4.2
17.4	1055	3.7	459	5.1	26.9	38.2	93	0	-5.1	-3.7
26.9	1691	11.4	792	0	38.4	24.6	217	0	0	-11.4
0	0	0	0	0	4	5.1	50	0	0	0
0	0	0	0	0	0	2.3	42	0	0	0
8.7	237	3	72	0	2	11.9	40	0	0	-3
8	77	1.3	39	0	0.9	3.4	14	0	0	-1.3
34.9	0	0	0	0	0	0	0	0	0	0
14.7	0	0.5	74	0	0	0	5	0	0	-0.5
9	0	10.3	244	0	1.9	7.5	146	0	0	-10.3
6.4	11	0.4	7	0.2	0.2	0.4	3	0	-0.2	-0.4

#### Fig.1 The Transpose of A for Standard Form

Here is the principle behind:

if  $\sum_{m=1}^{i} Nm \le k$ , then  $(\sum_{m=1}^{i} Nm \le k) \ge -k$ To put it simple in this case:

 $A(\text{mask},:) x \leq b_u = -A(\text{mask},:) x \geq -b_u,$ 

So that we could incorporate A(mask,:)  $x \le b_u$  into our single  $Ax \ge b$ 

b is 11 by 1, the first 9 rows are the annual lowest requirements of all the nutrition. The last 2 columns are the negative of annual highest limitations of vitamin-A and calcium.

c is 77 by 1, all elements are '1', since the nutrition content of each food in the data is calculated by buying \$1 of that food.

Finally, the standard form is obtained.

### 4.3 Getting the canonical form

#### Minimize $\hat{c}^T x$

Subject to  $\widehat{A}x \ge b, x \ge 0$ 

We introduce 11 slack variables. The original A is added a 11 by 11 negative identity matrix and becomes , which is 11 by 88.

*b* remains unchanged.

c is added with a 11 by 1 zero matrix to cope with the slack variables. This is exactly our, an 88 by 1 matrix. 4.4 Running Simplex Method

We use these , , and to run the simplex method program in MATLAB.

# 5. Result

# 5.1 Optimized result

LP optimal reached at objective = \$38.9486 (compare with \$39.93)

\$10.899 wheat flour \$0.72522 beef liver \$4.6373 cabbage \$0.6249 spinach \$22.062 navy beans

# 5.2 Four points that are note-worthy

1. \$38.9486 is the minimum cost of annual diet. Comparing to \$39.93 which Stiglar obtained without the use of simplex method, we have made it \$1 cheaper even with the upper bound of calcium and vitamin A.

2. Among 77 food types, only 5 are in our final food choice.

3. Stiglar's final food choice included evaporated milk which we don't. It didn't include beef liver which we do.

4. We have tried not taking the upper bound of calcium and vitamin A into consideration. Unexpectedly, we

acquire a completely identical result. The upper bound is not affecting our final result in this case.

# 6. Limitations

# 6.1 Taste Consideration

As we have discussed in the previously, the best diet distribution has been calculated by simplex method. That's how Steering Group (2010) completed the research Food for Tomorrow, Proposal for Finland's National Food Strategy similarly [7]. However, the distribution for received food does not consider the taste of actual dishes, which require condiments such as salt, sugar, monosodium glutamate, different sauces, soy, and oil. These added condiments can incorporate extraneous energy sources in our requirements and reduce the demands for typical food. So, in a practical perspective, condiments will increase the nutritional value of planned dishes and change the costs then. As general population are improving their economic conditions, condiments such as caviar and balsamic vinegar will increase food costs tremendously. In fact, these condiments will fulfill nutritional values by a small amount, but the current market does not allow it. In general, current population will choose expensive food for better taste instead of basic nutritional ingredients [8].

# 6.2 Unrealistic

In reality, the price of food varies and the general tendency is biased towards higher price because of inflation. Meanwhile, the government interference can result in a lower price than normal one. So, the cost data in our model are fixed while the practical cost can be influenced by distinctive economic developments and specific events. For instance, the Covid-19 pandemic has forced the food resources to be especially expensive and citizens will cost several times the original price to get fresh food with social distancing. Then, the food resources such as fast foods, food delivery, and instant noodles will be popular and preferred though most of them can hardly cover the required energy sources and nutrients. So, our diet optimization results will not cover and follow social tendencies as time goes by [9].

# 6.3 Data inconsistency

The daily lowest allowance of Vitamin-A is 5000 in the paper, but 2000 in our research. The comparison becomes less meaningful. No more for explanation, this limitation only exists in our model when compared with the original one. Meanwhile, though we considered large amount of food types, these data still came from a typical market at a chosen time. However, new food types have been created while traditional products such as wheat and rice have decreased their costs by advanced technology and labors [10].

#### 6.4 Present food waste and resource surplus

Although our central topic is to optimize the ratio between cost and food nutrients, the current global problem seems to be overwhelming food waste and resources surplus. About one billion tons of food is wasted every vear and the energy resources have a severe surplus over the nutritional requirements. So, the main issue here is to distribute the surplus part to those poor areas with food shortage. For our diet optimization problem, it is authentically awkward. On the one hand, the rich areas care costs little but waste food seriously. They can access the cheapest food resources easily. On the other hand, the poor areas do not own sufficient resources for this diet distribution and they often have difficulty to find a safe, healthy, and cheap food resource. In conclusion, though we designed a completer diet that optimized nutritional values, this diet may not be available in different regions [11].

# 6.5 Individual preference, allergy, and health limitations

Based on Garnett(2011,2014)'s argument in his articles, the diet optimized distribution can only determine the basic food types that a typical person with no desire and specific requirement would choose. Nevertheless, a realistic person will have specific preferences and food avoidance for health. Due to Food and Agriculture Organization's article, considering the condiments problem, if we incorporate the nutrients in the added condiments and take them into calculation, there will still be a result that seemed to be optimized. But the standard of an individual diner is distinctive. Meanwhile, the nutritional requirements are diversified severely because we have different bodies and organ conditions. Somebody may be thin, short, and weak, while others may be plump, high, and strong. Typically, our genes decide the allergies that a single person will have, which means they have to avoid the resulted food calculated by our model. Eventually, for every person who has this demand, they have to put their unique data into our model to get a personal food distribution [12].

# 7 Conclusion

In the final, this paper uses the model we created to solve the minimum cost diet problem by linear programming successfully. This paper has addressed some poor people's concern of not getting enough money to reach nutritional requirement of human body, at least in some extent. Comparing our results to 'The cost of subsistence'[4], we have made a cheaper minimum cost. While Stigler did the optimization by hand, we do this by linear programming, with the aid of computer. It is conspicuous to see how new algorithm can help solving practical problem. Also, it is found that even we don't take the upper limitation of Vitamin A and Calcium into account, the minimum cost doesn't change. This might be due to the fact that most food rich in other nutrients usually have a limited content of these two substances. However, in our opinion, this research is not perfect. It still contains various limitations mentioned in our paper, so it still has space for further modifications and improvements. For instance, the data are too old and the cost of condiments are not taken into account. Similar research methodology could therefore be applied to nowadays conditions, so that we could obtain the minimum annual cost of diet of a person in 2021. This would be very useful when natural or men-made disaster occur in the future, when everyone need to wisely allocate their expenditure to make a live. Therefore, even if we are living in today's modern society, these researches have its value and meaning.

# Reference

[1] Esra Base, Department of Industrial Engineering, A robust optimization approach to diet problem with overall glycemic load as objective function, 2014

[2] J. Mulvey, R. Vanderbei, S. Zenios, Robust optimization of large-scale systems, 1995

[3] Sual I. Gass, Linear Programming-Methods & Applications(Fifth Edition)

[4] George J. Stigler (1945), Agricultural & Applied Economics Association, The Cost of Subsistence

[5] Piste Pravina\*, Didwagh Sayaji and Mokashi Avinash, P.G .Department of Chemistry, Y. C. Institute of Science, Calcium and its Role in Human Body

[6] M. Shane Chapman, MD, Vitamin A: History, Current Uses, and Controversies, 2012

[7] Steering Group. Food for Tomorrow, Proposal for Finland's National Food Strategy; Minister of Agriculture and Forestry: Helsinki, Finland, 2010

[8] Health Council. Nutritional Guidelines for Energy, Protein, Fats and Digestible Carbohydrates (In Dutch); Health Council of the Netherlands: The Hague, The Netherlands, 2001

[9] Maillot, M.; Darmon, N.; Drewnowski, A. Are the lowestcost healthful food plans culturally and socially acceptable? Public Health Nutr. 2010, 13, 1178–1185.

[10] National Institute for Public Health and the Environment (RIVM). NEVO-table: Dutch Nutrient Database (in Dutch); RIVM: Bilthoven, The Netherlands, 2011

[11] World Health Organization (WHO). Diet, Nutrition and the Prevention of Chronic Diseases; WHO Technical Report Series; WHO: Geneva, Switzerland, 2003.

[12] European Commission (EC). Climate Change Factsheet; European Commision: Brussels, Belgium, 2014