ISSN 2959-6130

Comparison and Application of Markowitz Model and Index Model in Capital Markets

Qi Mu

Abstract:

The Markowitz Model represents a portfolio's returns for given risk levels. This research will imply the MM in Excel using the Solver tool. The model focuses on creating a portfolio that maximizes returns for a given level of risk or minimizes risks for a given level of return. Implementing the index model simplifies the calculation of expected returns and the covariance matrix of returns compared to the MM. This study aimed to compare the Markowitz Model and Index Model in constructing optimal portfolios from 10 stocks, the SPX index, and the 1-month federal funds rate, considering 20 years of monthly return data. This paper analyzes the impact of five constraints on these models' efficient frontiers, using Excel Solver for optimization. The study aims to understand how these models perform under various practical constraints and draw insights relevant to real-life portfolio management scenarios.

Keywords: Markowitz Model, Index Model, SPX Index, Optimal portfolio, Capital Markets

Introduction

Markowitz Model: Developed by Henry Markowitz in 1952 for constructing investment portfolios aiming to maximize expected return for a given level of risk or minimize risk for a given level of expected return. MM had a profound impact on finance and investment strategies. It forms the foundation for most quantitative approaches to portfolio optimization and has influenced the development of many financial products and investment strategies.

Index Model: The Index Model simplifies estimating the returns and risks of stocks in a portfolio. The Index Model is widely used for risk assessment and portfolio management.

SPX Index: The SPX index, also known as the S&P 500 Index, is a stock market index that measures the stock performance of 500 of the largest companies listed on stock exchanges in the United States. As one of the world's most watched stock market indices, the SPX is often used as a proxy for the health of the U.S. economy.

Optimal portfolio: An Optimal portfolio is a concept from investment theory, particularly Modern Portfolio Theory. The fundamental principle behind an optimal portfolio is the trade-off between risk and return. Investors need to balance their desire for the highest possible return against the level of risk they are willing to accept.

Data Selection:

The data utilized for analysis is sourced from Bloomberg Professional and Yahoo Finance. The dataset comprises the daily stock prices of 10 companies spanning a period from May 11, 2011, to May 12, 2021. The dataset includes the corresponding S&P 500 index values and the risk-free interest rate, specifically the one-month federal funds rate. ADBE, IBM, SAP, BAC, C, WFC, TRV, LUV, ALK, and HA are these companies. They can be divided into three different sectors.

Company name	Full name	introduction	sector
ADBE	Adobe Inc.	Adobe Inc., formerly Adobe Systems Incorporated, is an American multinational computer software company incorporated in Delaware and headquartered in San Jose, California. It has historically specialized in software for the creation and publication of a wide range of content	Technology

IBM	International Bussiness Machines Corporate	The International Business Machines Corporation is an American multinational technology corporation headquartered in Armonk, New York, and is present in over 175 countries. It specializes in computer hardware, middleware, and software and provides hosting and consulting services in areas ranging from mainframe computers to nanotechnology.	Technology
SAP	SAP SE	SAP SE is a German multinational software company based in Walldorf, Baden-Württemberg. It develops enterprise software to manage business operations and customer relations. The company is the world's leading enterprise resource planning (ERP) software vendor.	Technology
BAC	Bank of America Corporation	The Bank of America Corporation (often abbreviated BofA or BoA) is an American multinational investment bank and financial services holding company headquartered at the Bank of America Corporate Center in Charlotte, North Carolina, with investment banking and auxiliary headquarters in Manhattan.	Financial services
C	Citigroup Inc.	Citigroup Inc., or Citi, is an American multinational investment bank and financial services corporation incorporated in Delaware and headquartered in New York City. The company was formed by the merger of Citicorp, the bank holding company for Citibank, and Travelers in 1998; Travelers was spun off from the company in 2002.	Financial services
WFC	Wells Fargo& Company	Wells Fargo & Company is an American multinational financial services company with a significant global presence. The company operates in 35 countries and serves over 70 million customers worldwide. According to the Financial Stability Board, it is a systemically important financial institution. It is considered one of the "Big Four Banks" in the United States, alongside JPMorgan Chase, Bank of America, and Citigroup.	Financial services
TRV	The Travelers Companies, Inc.	The Travelers Companies, Inc., commonly known as Travelers, is an American insurance company. It is the second-largest writer of U.S. commercial property casualty insurance and the sixth-largest writer of U.S. personal insurance through independent agents.[3][4][citation needed] Travelers is incorporated in Minnesota, headquartered in New York City and its largest office in Hartford, Connecticut.[2] It has been a Dow Jones Industrial Average component since June 8, 2009.	Financial services
LUV	Southwest Airlines	Southwest Airlines Co. is a major airline based in the United States and the world's largest low-cost carrier. It is headquartered in Dallas, Texas, and has scheduled service to 121 destinations in the United States and ten additional countries. As of 2018, Southwest carried more domestic passengers than any other United States airline. It is currently the third-largest airline in North America based on passengers flown.	Industrials
ALK	Alaska Air Groups, Inc.	Alaska Air Group is an American airline holding company based in SeaTac, Washington, United States. The group owns two certificated airlines, Alaska Airlines, a mainline carrier, and Horizon Air, a regional carrier. Alaska Airlines, in turn, wholly owns an aircraft ground handling company, McGee Air Services.	Industrials

HA	Hawaiian Holdings, Inc	Southwest Airlines Co. is a major airline based in the United States and the world's largest low-cost carrier. It is headquartered in Dallas, Texas, and has scheduled service to 121 destinations in the United States and ten additional countries. As of 2018, Southwest carried more domestic passengers than any other United States airline. It is currently the third-largest airline in North America based on passengers flown.	Industrials

Theoretic models

rp = w1r1 + w2r2

rp represents the rate of return on the portfolio. w1 and w2 are the weights of asset1 and asset2 in the portfolio, respectively. These weights indicate the proportion of each asset's total portfolio value. Typically, w1+w2=1 (or 100%), meaning all investment is distributed between these two assets.

r1 and r2 are the rates of return on asset1 and asset2, respectively. These are usually expressed as a percentage or a decimal fraction.

This equation calculates the portfolio's overall return as a weighted average of the returns of the individual assets. The weight of each asset reflects its relative importance or proportion in the overall portfolio.

E(rp) = w1E(r1) + w2E(r2)

E(rp) represents the expected return of the portfolio. wi and w2 are the weights of the different assets in the portfolio. These weights indicate how much of the portfolio is invested in each asset. They usually sum up to 100%. E(r1) and E(r2) are the expected returns of the individual assets in the portfolio. The expected return of the entire portfolio is a weighted average of the expected returns of the individual assets that make up the portfolio. The weights(w1 and w2) reflect how much of the total investments in each asset.

D(rp)=E((rp−E(rp))2)=E(rp)2−E(rp)2

D(rp) represents the variance of the portfolio's return. Variance is a statistical measure that describes the spread of the returns (how much the returns deviate from the average return.

E((rp−E(rp))2) calculated the expected value of the squared deviation of the portfolio's return (rp) from its expected return (E(rp))

E(*rp*2) is the expected value of the square of the portfolio's return.

E(rp)2 is the square of the portfolio's expected return.

The optimal portfolio

After finding all those values, figure out the optimal portfolio by identifying the set of available assets to include. These could be stocks, bonds, commodities, etc. Calculate the expected return, which is often based on historical data. Determine the risks associated with each asset. Risk is typically measured as the variance or standard deviation of the asset's returns. How each pair of assets in a portfolio interacts with each other can be measured through correlation. This step is crucial because diversification benefits depend on these inter-asset relationships. The optimization problem could be maximizing return for a given level of risk for a given level of return. These include budget constraints, risk tolerance levels, and other constraints specific to the investor. The optimization problem can be solved using various methods. The goal is to find the asset weightings that give the best trade-off between the expected return and risk. The Efficient Frontier represents a set of optimal portfolios that offer the highest expected return for a defined level of risk or the lowest risk for a given level of expected return.

Normality Test

We employ fundamental statistical methods like probability density functions, basic statistics, and Q-Q plots to assess the daily and monthly data similarity for ten stocks to a normal distribution. This evaluation helps verify whether the model's assumption of normality holds.

1. We use Excel to calculate the relevant statistical descriptions of each stock's annual data; these are the results shown:

	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA
Annual Average Return	7.5%	19.6%	4.8%	12.0%	11.1%	1.0%	8.9%	9.1%	9.8%	17.4%	26.9%
Annual StDev	14.9%	31.8%	23.2%	33.9%	39.3%	42.5%	28.1%	20.0%	31.8%	37.7%	62.1%
beta	1.00	1.42	1.01	1.48	1.60	2.01	1.05	0.80	1.15	1.18	1.63
alpha	0.00	0.09	-0.03	0.01	-0.01	-0.14	0.01	0.03	0.01	0.09	0.15
residual Stdev	0.0%	23.8%	17.6%	25.8%	31.4%	30.3%	23.4%	16.0%	26.8%	33.4%	57.2%

From the table, we can see that HA has the highest return.

2. We use the correlation function to get the correlation among ten stocks; the return has shown:

	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA
SPX	100.0%	66.5%	64.9%	64.9%	60.2%	70.2%	55.5%	59.8%	53.7%	46.4%	39.0%
ADBE	66.5%	100.0%	45.5%	53.4%	42.3%	46.3%	29.8%	45.2%	38.8%	23.3%	18.0%
IBM	64.9%	45.5%	100.0%	58.5%	31.3%	42.0%	26.7%	38.2%	34.7%	35.7%	24.6%
SAP	64.9%	53.4%	58.5%	100.0%	33.1%	43.4%	29.8%	37.5%	31.8%	28.2%	14.4%
BAC	60.2%	42.3%	31.3%	33.1%	100.0%	82.6%	76.1%	39.3%	42.8%	27.5%	33.8%
C	70.2%	46.3%	42.0%	43.4%	82.6%	100.0%	70.3%	51.2%	42.8%	30.4%	34.3%
WFC	55.5%	29.8%	26.7%	29.8%	76.1%	70.3%	100.0%	34.5%	40.6%	34.7%	35.8%
TRV	59.8%	45.2%	38.2%	37.5%	39.3%	51.2%	34.5%	100.0%	40.7%	36.0%	24.0%
LUV	53.7%	38.8%	34.7%	31.8%	42.8%	42.8%	40.6%	40.7%	100.0%	51.9%	42.2%
ALK	46.4%	23.3%	35.7%	28.2%	27.5%	30.4%	34.7%	36.0%	51.9%	100.0%	40.4%
HA	39.0%	18.0%	24.6%	14.4%	33.8%	34.3%	35.8%	24.0%	42.2%	40.4%	100.0%

Based on the result, there is a strong correlation between BAC and C; by contrast, the correlation between HA and SAP is the weakest.

Markowitz Model Results

Constraint 1:

Regulation T allows investors to use borrowed funds up to 50% of their account equity for investments. In portfolio optimization, this rule should be included as a constraint, ensuring that the portion of the portfolio funded by borrowing does not exceed this limit.

The portfolio is satisfied with minimum variance; in accordance with the minimum variance requirement, the portfolio generates an annual return of 6.72% with a corresponding portfolio of 0.1175 and a sharp ratio of 0.572.

The portfolio is satisfied with the maximum sharp ratio; in accordance with the maximum sharp ratio, the portfolio generates an annual return of 17.59% with a corresponding portfolio of 0.1770 and a sharp ratio of 0.994.

MM (Constr1):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe			
MinVar	110.97%	-9.68%	5.14%	-9.90%	0.35%	-22.54%	14.06%	19.45%	-0.13%	-4.86%	-2.85%	6.72%	11.75%	0.572	CAL:	0.0%	0.0%
MaxSharpe	39.54%	28.26%	-0.10%	0.29%	16.67%	-48.07%	19.80%	30.14%	-1.80%	7.70%	7.57%	17.59%	17.70%	0.994	2.5	44.0%	44.2%

Constraint 2:

The additional optimization constraint mentioned is designed to imitate client-specified “box” constraints on weights. Its purpose is to provide a means to replicate such weight restrictions.

The portfolio is satisfied with a minimum variance; in accordance with the minimum variance requirement, the portfolio generates an annual return of 6.97% with a corresponding portfolio of 0.1179 and a Sharpe ratio of 0.591.

The portfolio is satisfied with the maximum sharp ratio; by the maximum Sharpe ratio, the portfolio generates an annual return of 22.07% with a corresponding portfolio of 0.2133 and a Sharpe ratio of 1.035.

MM (Constr2):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	100.00%	-8.53%	8.25%	-9.24%	0.66%	-23.03%	16.19%	22.93%	0.28%	-4.76%	-2.75%	6.97%	11.79%	0.591
MaxSharpe	50.09%	35.81%	-22.32%	7.19%	29.74%	-66.14%	20.56%	35.57%	-14.83%	13.45%	10.87%	22.07%	21.33%	1.035

Constraint 3:

The portfolio is satisfied with a minimum variance; by the minimum variance requirement, the portfolio generates an annual return of 6.69% with a corresponding portfolio of 0.1175 and a Sharpe ratio of 0.591. The portfolio is satisfied with the maximum sharp ratio; by the maximum Sharpe ratio, the portfolio generates an annual return of 22.07% with a corresponding portfolio of 0.2133 and a Sharpe ratio of 1.035.

MM (Constr3):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	111.46%	-9.91%	5.27%	-10.07%	0.60%	-22.81%	14.06%	19.53%	-0.26%	-4.98%	-2.88%	6.69%	11.75%	0.570
MaxSharpe	50.09%	35.81%	-22.32%	7.19%	29.74%	-66.14%	20.56%	35.57%	-14.83%	13.45%	10.87%	22.07%	21.33%	1.035

Constraint 4:

The portfolio is satisfied with a minimum variance; the minimum variance requirement generates an annual return of 7.79% with a corresponding portfolio of 0.1461 and a Sharpe ratio of 0.533.

The portfolio is satisfied with the maximum sharp ratio; by the maximum Sharpe ratio, the portfolio generates an annual return of 18.24% with a corresponding portfolio of 0.2475 and a Sharpe ratio of 0.737.

MM (Constr4):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	83.61%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	16.39%	0.00%	0.00%	0.00%	7.79%	14.61%	0.533
MaxSharpe	0.00%	50.27%	0.00%	0.00%	0.00%	0.00%	0.00%	17.56%	0.00%	19.52%	12.66%	18.24%	24.75%	0.737

Constraint 5:

The portfolio is satisfied with a minimum variance; by the minimum variance requirement, the portfolio generates an annual return of 9.38% with a corresponding portfolio of

0.1545 and a sharp ratio of 0.607.
The portfolio is satisfied with the maximum sharp ratio; by the maximum Sharpe ratio, the portfolio generates an annual return of 26.53% with a corresponding portfolio of 0.2598 and a Sharpe ratio of 1.021.

MM (Constr5):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	0.00%	3.53%	34.20%	-1.99%	1.18%	-24.94%	34.79%	52.68%	5.03%	-2.86%	-1.61%	9.38%	15.45%	0.607
MaxSharpe	0.00%	50.82%	-19.51%	13.81%	36.74%	-76.96%	28.73%	49.95%	-16.54%	18.45%	14.50%	26.53%	25.98%	1.021

IM Models

constraint 1: According to the data. The portfolio is satisfied with maximum sharp ratio and minimum

M (Constr1):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	138.69%	-9.85%	0.03%	-9.68%	-8.19%	-15.39%	-0.28%	11.23%	-2.17%	-1.77%	-2.62%	6.07%	11.96%	0.508
MaxSharpe	48.18%	35.88%	-14.54%	2.04%	-0.61%	-34.85%	3.97%	28.36%	3.60%	17.73%	10.24%	18.90%	21.04%	0.898

Constraint 2: According to the data. The portfolio is satisfied with maximum sharp ratio and minimum variance:

M (Constr2):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	100.00%	-6.58%	8.50%	-6.99%	-6.49%	-13.97%	3.74%	23.09%	0.74%	0.08%	-2.12%	6.90%	12.47%	0.553
MaxSharpe	54.94%	38.26%	-22.71%	2.98%	-2.31%	-37.58%	4.25%	28.63%	4.00%	18.66%	10.89%	19.81%	21.99%	0.901

Constraint 3: According to the data. The portfolio is satisfied with maximum sharp ratio and minimum variance:

IM (Constr3):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	144.39%	-10.70%	-0.63%	-10.37%	-8.62%	-15.69%	-1.35%	10.96%	-2.97%	-2.27%	-2.75%	5.85%	11.95%	0.490
MaxSharpe	54.94%	38.26%	-22.71%	2.98%	-2.31%	-37.58%	4.25%	28.63%	4.00%	18.66%	10.89%	19.81%	21.99%	0.901

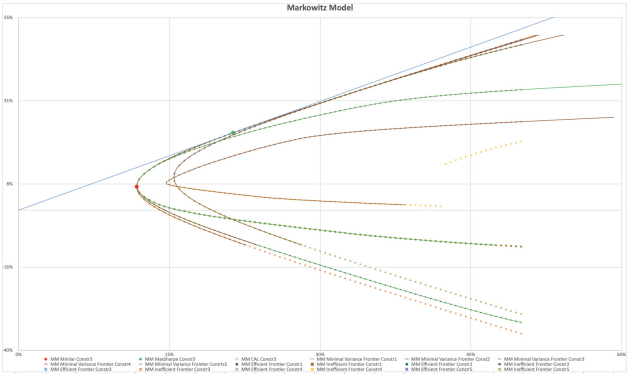
Constraint 4: According to the data. The portfolio is satisfied with maximum sharp ratio and minimum variance:

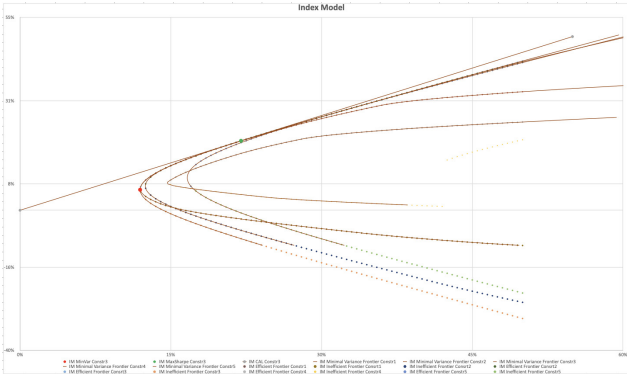
IM (Constr4):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	83.61%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	16.39%	0.00%	0.00%	-0.00%	7.79%	14.61%	0.533
MaxSharpe	0.00%	40.84%	0.00%	0.00%	0.00%	0.00%	0.00%	26.27%	0.00%	20.52%	12.38%	17.28%	23.35%	0.740

Constraint 5: According to the data. The portfolio is satisfied with maximum sharp ratio and minimum variance:

IM (Constr5):	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA	Return	StDev	Sharpe
MinVar	0.00%	2.68%	29.08%	0.61%	-1.69%	-10.08%	15.22%	50.40%	9.09%	5.38%	-0.70%	9.26%	16.64%	0.556
MaxSharpe	0.00%	52.54%	-20.56%	8.64%	0.79%	-41.18%	9.50%	42.11%	8.47%	25.23%	14.46%	23.79%	26.68%	0.892

Comparative Analysis





Constraint 1: The efficient frontiers of the Markowitz model and the index model differ significantly, with the efficient frontier of the IM model located below that of the Markowitz model, which suggests that under the same investment constraints 1, the risk-return performance of the portfolio constructed using the IM model is inferior to using the Markowitz Model.
Constraint 2: The efficient frontiers of the Markowitz model and the index model differ significantly, with the efficient frontier of the IM model located below that of the Markowitz model, which suggests that under the same investment constraints 2, the risk-return performance of the portfolio constructed using the IM model is inferior to using the Markowitz Model.
Constraint 3: The efficient frontiers of the Markowitz model and the index model differ significantly, with the efficient frontier of the IM model located below that of the Markowitz model, which suggests that under the same investment constraints 3, the risk-return performance of the portfolio constructed using the IM model is inferior to using the Markowitz Model.
Constraint 4: The efficient frontiers of the Markowitz model and the index model are different, with the efficient frontier of the IM model located below that of the Markowitz model, which suggests that under the same

investment constraints 4, the risk-return performance of the portfolio constructed using the IM model is inferior to using the Markowitz Model.

Constraint 5: The efficient frontiers of the Markowitz and index models differ. The efficient frontier of the IM model is located below that of the Markowitz model, which suggests that under the same investment constraint 5, the risk-return performance of the portfolio constructed using the IM model is inferior to that using the Markowitz Model.

Analysis

The Markowitz model is always more efficient than the IM model in all five constraints. When comparing the Modern Portfolio Theory (MPT) developed by Harry Markowitz with the approaches used in Investment Management (IM) models, the Efficient Frontier in Markowitz's model often positions itself higher than that in IM models. This is due to distinct differences in these two frameworks' underlying assumptions and constraints.

Markowitz's Model emphasizes the principle of diversification, which plays a critical role in portfolio optimization. This model stresses the importance of understanding and leveraging the correlation between various assets, allowing for the construction of portfolios that offer higher returns for a specified level of risk or reducing risk for a given return. This is in contrast to the IM approach, which may not fully harness the potential of diversification, especially if the model imposes more stringent constraints on asset allocation or fails to adequately consider asset correlations.

Another difference between these models is the approach towards asset selection and portfolio constraints. Markowitz's model typically embraces a wider array of assets and imposes fewer limitations on portfolio allocation. This flexibility allows for a more comprehensive optimization process, often resulting in portfolios that more efficiently balance risk and return, as depicted on the Efficient Frontier. In contrast, the IM model may restrict the selection of assets or apply more rigid constraints, leading to a less optimized risk-return trade-off.

The differences between the Efficient Frontiers of Markowitz's Model and IM models can be attributed to their divergent approaches to diversification, asset selection, and risk management. These disparities highlight the varied methodologies and theoretical underpinnings inherent in financial portfolio management, each with its advantages and limitations in the context of investment strategy formulation.

References

- "Adobe Inc. 2022 Annual Form 10-K Report". *Adobe*. December 2, 2022. Archived from the original on May 2, 2023. Retrieved April 3, 2023.
- "Certificate of Incorporation of Computing-Tabulating-Recording-Co", *Appendix to Hearings Before the Committee on Patents, House of Representatives, Seventy-Fourth Congress, on H. R. 4523, Part III*, United States Government Printing Office, 1935 [Incorporation paperwork filed June 16, 1911], archived from the original on August 3, 2020, Retrieved July 18, 2019
- "SAP Q4 2022 Quarterly Statement" (PDF). SAP SE. Retrieved 4 February 2023.
- "in PDF on page 2 (Assets Under Management)" (PDF). bankofamerica.com.
- "US SEC: 2022 Form 10-K Citigroup Inc". U.S. Securities and Exchange Commission. February 27, 2023.
- Wack, Kevin (February 26, 2020). "How New York became Wells Fargo's new center of power." *American Banker*.
- "The Travelers Companies, Inc. 2022 Annual Report (Form 10-K)". *sec.gov*. U.S. Securities and Exchange Commission. 16 February 2023.
- "Southwest Airlines Announces New Crew Base for Pilots and Flight Attendants at Nashville International Airport (BNA)" (Press release). August 14, 2023. Retrieved October 26, 2023.
- Alaska Air Group, Inc. 2022 Annual Report (Form 10-K)". U.S. Securities and Exchange Commission. February 13, 2023.
- "Airline Certificate Information – Detail View." *av-info.faa.gov*. Federal Aviation Administration. December 27, 1946. Archived from the original on January 20, 2018. Retrieved January 19, 2018. Certificate Number HALA005A