Modern Portfolio Theory and Its Applications

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Abstract:

According to the modern portfolio theory, the monthly return data of 4 groups of portfolios (40 stocks) from November 5, 2011, to November 5, 2023, are empirically found: (1) the average monthly yield curve of stock portfolios group#1 and group#4 fluctuates significantly, while the volatility of equity portfolios group#2 and group#3 is relatively moderate. Especially after the new crown epidemic outbreak in 2020, the average monthly returns of all portfolios fluctuated sharply. (2) The expected return of NVDA is the highest among all stocks in the portfolio, and the IMB is the lowest. (3) Compared to the given portfolio returns, the #3 portfolio performs best, while the fund of group #1 performs the worst, and the remaining two portfolios perform well. In addition, if only the portfolio return and standard deviation are used as the portfolio return risk measures, the theoretical portfolio is better than the existing actual portfolio. Therefore, the return of a portfolio of securities will not exceed the yield of the benchmark portfolio at the same level of risk, which, to some extent, supports the hypothesis of the effectiveness of the securities market.

Keywords: Portfolio;Expected rate of return;risk;Market Effectiveness

1. Introduction

As one of the core research contents of finance, asset portfolios have become an important part of the modern investment field. When Markowitz proposed the mean-variance theory, portfolio theory ushered in a very active development and has been widely used in the field of asset investment[1]. With the rapid development of information technology and the increasing improvement of the securities system, “effective investment” under the support of market maturity has become an irreversible development trend of asset management.[2].

2. Portfolio models and empirical analysis

2.1 Markowitz Portfolio Model

Based on mathematical methods, Markowitz proposed a mean-variance portfolio model based on the assumptions that asset returns follow a multivariate normal distribution and that investors are risk-averse and follow the principle of investment diversification for efficient investment. The specific mathematical form is as follows in equation (1):

$$\begin{aligned} &\min \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} = X^{'} V X \\ &ns.t: R_p = \sum_{i=1}^{n} x_i R_i = R^{'} X = r \\ &\sum_{i=1}^{n} x_i = I^{'} X = 1 \quad x_i \geqslant 0 (i = 1, \cdots, n) \end{aligned} \tag{1}$$

Where $\sigma_p$is the standard deviation of the portfolio; R_p is the expected rate of return of the portfolio; R_i is the expected rate of return of $i^{'}s$investment vehicle; $x_i$is the weight that $i^{'}s$investment vehicle has in a portfolio; n is the number of investment instruments in the portfolio; $r$is the expected rate of return of investors; σ_{ij} is the covariance between $i^{'}s$investment vehicle and$j^{'}s$ investment vehicle; $V$is the covariance matrix of each risk asset in the portfolio; R is the average of the returns on each risk asset in the portfolio; X is the proportion of each risky asset in the portfolio; I is a vector with all components of 1.

2.2 Empirical analysis

2.2.1 Selection of research objects and sample intervals

(1) Research object: According to the requirements of the course document, the research object is determined to be divided into four groups (a total of 40 stocks). Figure 1 shows the details.

	Group #1	Group #2	Group #3	Group #4
Index	SPX	SPX	SPX	SPX
Stock #1	ADBE	AMZN	NVDA	QCOM
Stock #2	IBM	AAPL	CSCO	AKAM
Stock #3	SAP	CTXS	INTC	ORCL
Stock #4	BAC	JPM	GS	MSFT
Stock #5	C	BRK/A	USB	CVX
Stock #6	WFC	PGR	TD CN	XOM
Stock #7	TRV	UPS	ALL	IMO
Stock #8	LUK	FDX	PG	KO
Stock #9	ALK	JBHT	JNJ	PEP
Stock #10	HA	LSTR	CL	MCD
Risk-free rate	FEDL01	FEDL01	FEDL01	FEDL01

Fig.1 Research object

(2) Selection of sample intervals: According to the requirements of the course documents, the sample intervals are set from November 5, 2011, to November 5, 2023. The return value of individual stocks is set to the monthly return.

2.2.2 Methods for dealing with basic indicators

(1) Calculate the average monthly rate of return of the portfolio. First, calculate the weekly return of the individual stocks in each portfolio[3]. The calculation is as follows in equation (2):

$$r_{j,t} = \frac{P_{j,t} - P_{j,t-1}}{P_{j,t-1}} \tag{2}$$

In Eq. (2), $r_{j,t}$is the $t$week's return of the stock j ($j = 1,2,3,4, t = 1,2,\cdots 52$), $P_{j,t-1}$is the adjusted yield of the stock$j$ in the $t-1$week.

Second, the average monthly return of the portfolio is obtained by weighting the average return of individual stocks in each portfolio. The calculation is as follows in equation (3):

$$R_m = \sum r_{j,t} / m; (m = 1,2,3,4) \tag{3}$$

where is the portfolio's average monthly rate of return$m-th$; $m$is the number of combinations.

(2) Calculate the risk of each portfolio. First, calculate the variance of the sample return of the stock[4]. The calculation is as follows in equation (4):

$$\mathrm{var}(r_i) = \frac{\sum_{i=1}^{n} (r_{it} - \bar{r}_i)^2}{(N-1)} \tag{4}$$

Second, calculate the sample return covariance for stocks$i$ j. The calculation is as follows in equation (5):

$$\mathrm{cov}(r_i, r_j) = \frac{\sum_{i,j=1}^{N} (r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j)}{(N-1)} \tag{5}$$

2.2.3 Result analysis

(1) The average monthly rate of return of each portfolio. Calculate the average monthly returns of the four portfolios according to the formula and draw a line chart 2. The average monthly yield curve of equity portfolios #1 and #4 fluctuates significantly, while equity portfolios #2 and #3 fluctuate more modestly. Especially after the new crown epidemic outbreak in 2020, the average monthly returns of all portfolios fluctuated sharply.

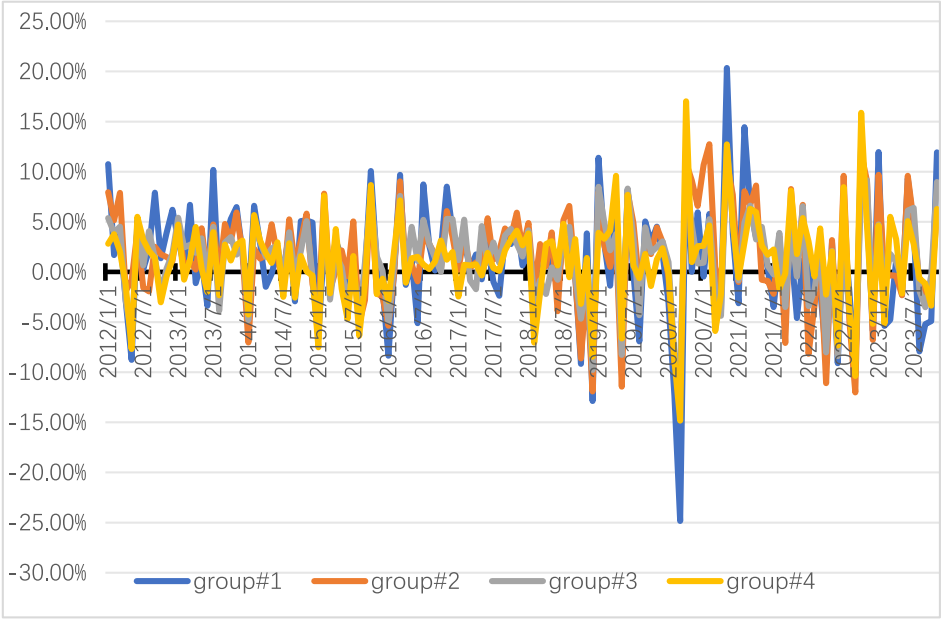

Figure 2 The average monthly return of the portfolio

(2) The expected rate of return of individual stocks. Table 1 shows the expected returns of all stocks in the four portfolios and finds that the individual stocks in all portfolios have the highest expected returns in NVDA and the lowest in IMB. The highest expected return for BAC in Portfolio 1 is 0.0173, and the lowest is 0.0042 for IMB. The highest value for AMZN in Portfolio 2 is 0.0237, and the lowest value for UPS is 0.0101. The highest NVDA in Portfolio 3 is 0.0431, and the lowest CL is 0.0066. The highest MSFT in Portfolio 4 is 0.0224, and the lowest KO

is 0.0072.

Table 1 Portfolio expected rate of return

Group#1	ADBM	IMB	SAP	BAC	C	WFC	TRV	LUK	ALK	HA
	0.0247	0.0042	0.0119	0.0173	0.0097	0.0084	0.0112	0.0071	0.0112	0.0096
Group#2	AMZN	AAPL	CTRN	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR
	0.0237	0.0225	0.0207	0.0158	0.0120	0.0192	0.0101	0.0122	0.0127	0.0118
Group#3	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL
	0.0431	0.0119	0.0097	0.0140	0.0073	0.0083	0.0145	0.0092	0.0092	0.0066
Group#4	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
	0.0130	0.0125	0.0138	0.0224	0.0081	0.0073	0.0085	0.0072	0.0097	0.0105

(3) the variance-covariance matrix of the portfolio. Tables respectively.
2-5 show group #1-group#4 variance-covariance matrix

Table2 The variance-covariance matrix of the portfolio of Group#1

	ADBM	IMB	SAP	BAC	C	WFC	TRV	LUK	ALK	HA
ADBM	0.0058	0.0016	0.0032	0.0027	0.0031	0.0016	0.0011	0.0012	0.0018	0.0024
IMB	0.0016	0.0039	0.0019	0.0025	0.0027	0.0017	0.0013	0.0009	0.0021	0.0024
SAP	0.0032	0.0019	0.0055	0.0027	0.0032	0.0019	0.0012	0.0019	0.0020	0.0024
BAC	0.0027	0.0025	0.0027	0.0081	0.0069	0.0051	0.0021	0.0015	0.0049	0.0064
C	0.0031	0.0027	0.0032	0.0069	0.0084	0.0050	0.0026	0.0020	0.0053	0.0067
WFC	0.0016	0.0017	0.0019	0.0051	0.0050	0.0055	0.0020	0.0008	0.0045	0.0052
TRV	0.0011	0.0013	0.0012	0.0021	0.0026	0.0020	0.0029	0.0001	0.0026	0.0030
LUK	0.0012	0.0009	0.0019	0.0015	0.0020	0.0008	0.0001	0.0115	0.0017	0.0017
ALK	0.0018	0.0021	0.0020	0.0049	0.0053	0.0045	0.0026	0.0017	0.0109	0.0103
HA	0.0024	0.0024	0.0024	0.0064	0.0067	0.0052	0.0030	0.0017	0.0103	0.0217

Table 3 The variance-covariance matrix of the portfolio of Group#2

	AMZN	AAPL	CTRN	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR
AMZN	0.0077	0.0035	0.0009	0.0016	0.0014	0.0008	0.0022	0.0026	0.0017	0.0011
AAPL	0.0035	0.0064	0.0032	0.0016	0.0014	0.0013	0.0020	0.0026	0.0019	0.0017
CTRN	0.0009	0.0032	0.0255	0.0043	0.0019	0.0020	0.0023	0.0033	0.0030	0.0030
JPM	0.0016	0.0016	0.0043	0.0051	0.0022	0.0010	0.0020	0.0028	0.0024	0.0022
BRK/A	0.0014	0.0014	0.0019	0.0022	0.0022	0.0008	0.0018	0.0021	0.0017	0.0015
PGR	0.0008	0.0013	0.0020	0.0010	0.0008	0.0032	0.0013	0.0016	0.0009	0.0009
UPS	0.0022	0.0020	0.0023	0.0020	0.0018	0.0013	0.0049	0.0040	0.0030	0.0023
FDX	0.0026	0.0026	0.0033	0.0028	0.0021	0.0016	0.0040	0.0068	0.0032	0.0026
JBHT	0.0017	0.0019	0.0030	0.0024	0.0017	0.0009	0.0030	0.0032	0.0044	0.0027
LSTR	0.0011	0.0017	0.0030	0.0022	0.0015	0.0009	0.0023	0.0026	0.0027	0.0035

Table 4 The variance-covariance matrix of the portfolio of Group#3

	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL
NVDA	0.0151	0.0027	0.0036	0.0033	0.0017	0.0025	0.0001	0.0001	0.0004	0.0002
CSCO	0.0027	0.0051	0.0021	0.0025	0.0013	0.0016	0.0012	0.0009	0.0009	0.0010
INTC	0.0036	0.0021	0.0059	0.0021	0.0011	0.0014	0.0008	0.0005	0.0011	0.0005
GS	0.0033	0.0025	0.0021	0.0065	0.0037	0.0028	0.0020	0.0007	0.0013	0.0008
USB	0.0017	0.0013	0.0011	0.0037	0.0048	0.0026	0.0017	0.0006	0.0007	0.0008
TDCN	0.0025	0.0016	0.0014	0.0028	0.0026	0.0032	0.0014	0.0006	0.0007	0.0008
ALL	0.0001	0.0012	0.0008	0.0020	0.0017	0.0014	0.0030	0.0008	0.0012	0.0011
PG	0.0001	0.0009	0.0005	0.0007	0.0006	0.0006	0.0008	0.0020	0.0010	0.0012
JNJ	0.0004	0.0009	0.0011	0.0013	0.0007	0.0007	0.0012	0.0010	0.0020	0.0011
CL	0.0002	0.0010	0.0005	0.0008	0.0008	0.0008	0.0011	0.0012	0.0011	0.0019

Table 5 The variance-covariance matrix of the portfolio of Group#4

	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
QCOM	0.0101	0.0023	0.0018	0.0026	0.0016	0.0014	0.0027	0.0010	0.0013	0.0012
AKAM	0.0023	0.0072	0.0018	0.0016	0.0011	0.0007	0.0018	0.0006	0.0004	0.0004
ORCL	0.0018	0.0018	0.0040	0.0019	0.0018	0.0018	0.0022	0.0009	0.0009	0.0013
MSFT	0.0026	0.0016	0.0019	0.0037	0.0013	0.0009	0.0014	0.0009	0.0009	0.0011
CVX	0.0016	0.0011	0.0018	0.0013	0.0055	0.0047	0.0057	0.0013	0.0013	0.0016
XOM	0.0014	0.0007	0.0018	0.0009	0.0047	0.0054	0.0056	0.0012	0.0010	0.0013
IMO	0.0027	0.0018	0.0022	0.0014	0.0057	0.0056	0.0099	0.0016	0.0012	0.0018
KO	0.0010	0.0006	0.0009	0.0009	0.0013	0.0012	0.0016	0.0021	0.0014	0.0013
PEP	0.0013	0.0004	0.0009	0.0009	0.0013	0.0010	0.0012	0.0014	0.0018	0.0012
MCD	0.0012	0.0004	0.0013	0.0011	0.0016	0.0013	0.0018	0.0013	0.0012	0.0022

On this basis, linear programming is carried out in the EXCEL[5]. Set the optimal shareholding ratio to be solved as a variable cell. The constraints are that the shareholding ratio is equal to 1, the portfolio return is equal to the average weekly return of the portfolio, and the planning equation can be solved under the non-negative assumption, so that the optimal shareholding ratio and total risk of the stock without short selling restrictions can be obtained.

(4) Performance evaluation of investment portfolios. Figure 2 maps the effective investment frontier of group #1. According to the portfolio theory, the performance of 4 portfolios is evaluated. Portfolio #3 performs best relative to a given portfolio return, while fund #1 performs the worst, with the remaining two portfolios performing well. In addition, if only the portfolio return and standard deviation are used as the portfolio return risk measures, the theoretical portfolio is better than the existing actual portfolio.

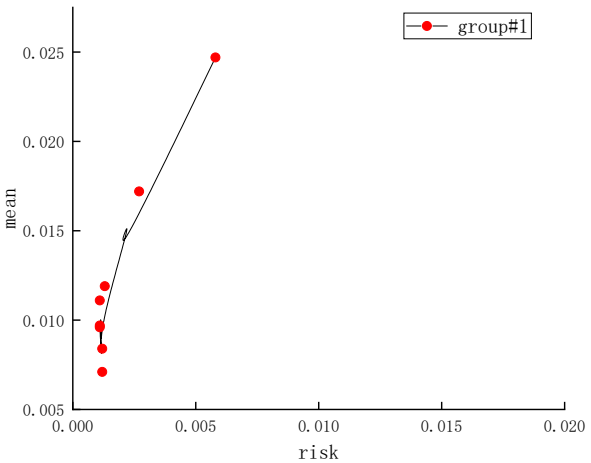

Figure 3 Efficient investment frontier curve for group #1.

3. discussion

In this paper, the weekly average return of the real portfolio is used as the limited interest rate to establish the portfolio theory, and the average weekly return of the actual portfolio may not be the full market benchmark interest rate portfolio. Foreign studies also show that under the same risk level, the return of the securities portfolio will not exceed the return of the market benchmark portfolio.[6], which to some extent supports the hypothesis of the effectiveness of the securities market.

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