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Portfolio optimization based on quadratic programming: a comparative risk-return analysis with mean-variance model under different time periods

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Abstract:

This paper compares the specific performance of Markowitz's mean-variance model and quadratic programming optimization model in different economic periods by means of empirical analysis. The paper compares and analyzes the return performance and risk assessment of the mean-variance model and the quadratic programming optimization model in different economic periods (including expansion, stabilization, recession, and recovery periods) by introducing various risk measurement tools, such as the lower semi variance, value-at-risk (VaR), and conditional value-at-risk (CVaR). The results of the comparative analysis show that the quadratic programming optimization model is more effective in controlling extreme risks and outperforms the mean-variance model during periods of high market volatility, while the difference in performance between the two models is small during periods of relative economic stability. By comparing the effective boundary images and Sharpe ratios of the quadratic programming model and the mean-variance model, this paper provides an effective reference basis for investors to choose appropriate optimization strategies in different economic periods.

Keywords: portfolio optimization, quadratic programming, mean-variance model, risk management, economic cycle

1. Introduction

In the modern financial market, investors will make decisions with the help of various portfolio optimization models to maximize their returns. In 1952, Harry Markowitz proposed the mean-variance model for the first time, which became the cornerstone of the modern portfolio theory, and influenced the evolution and development of the subsequent types of portfolio optimization models. The model aims to weigh the expected returns and risks through the effective combination of asset allocation, helping investors to realize the optimal solution of returns within the established risk tolerance range. However, as market volatility and complexity increase, such as the financial crisis in 2008 and the new crown epidemic in 2020, the mean-variance model exposes its limitations in the face of such extreme and asymmetric risks. This has prompted theoretical researchers and industry practitioners to seek more effective optimization methods.

To address the challenge, quadratic programming has been applied to optimize portfolios by combining more sophisticated risk measures such as lower half variance, CVaR, VaR, to improve portfolio performance under extreme risk scenarios. These risk metrics are introduced not only to focus on overall volatility, but also to effectively measure and manage potential losses in extreme situations.

In addition, different phases of the economic cycle, such as periods of economic expansion, plateau, recession and recovery, also have a significant impact on portfolio performance, which in turn influences investors' decisions. During expansionary periods, economic dynamism increases, market sentiment is optimistic, and investors tend to be willing to take on more risk, while during recessionary periods, economic activity weakens, GDP declines, unemployment rises, risk aversion increases in the market, and capital is withdrawn from risky assets. In view of this, exploring the performance of the quadratic programming-based portfolio optimization model versus the traditional Markowitz mean-variance model in different economic cycles is of significant practical value and can provide investors with a useful basis for decision-making in uncertain market environments.

In this paper, we will provide an empirical analysis to compare in detail the risk and return performance of a portfolio based on quadratic programming optimization with the Markowitz mean-variance model in different economic cycles. We will use risk management tools such as lower half variance, CVaR, and VaR to evaluate two models in different periods: economic expansion, recession, recovery, and stabilization. This paper will present both theoretical insights and practical guidance for investors in different economic conditions by comparing the validity boundary and Sharpe ratio.

2. Literature Review

Modern portfolio theory is anchored on Markowitz's Mean-Variance Model (MVM) dating back to 1952, which had dual objectives: to maximize return while simultaneously minimizing portfolio volatility. It became a major tool in asset allocation. However, as markets have become more complex and volatile, researchers have found that variance alone doesn't fully account for systematic risk, particularly during extreme market fluctuations. In those cases, the mean-variance model falls short.

These deficiencies have been further overcome by the development of higher-order risk measures such as Conditional Value-at-Risk (CVaR), Value-at-Risk (VaR), and Lower Half Covariance (LHC). The VaR, facilitated in the 1990s mainly by J.P. Morgan, showed the maximum possible loss at a specific confidence level but itself was not capable of capturing the risks lying beyond its threshold. Then, Rockafellar and Uryasev 2020 introduced the CVaR, which works much better on extreme tail risks and is now widely used in portfolio optimization [1].

Also, quadratic programming approaches have emerged as important approaches toward the solution of multi-objective optimization problems. All these in incorporating such risk measures as VaR, CVaR, and lower half variance into their methods to increase precision in portfolio optimization. Estrada (2008) cited that the lower half-variance is a good risky volatility measure to capture the negative volatility risk when returns realized were lower than expectations [2]. Fabozzi et al. (2007) have also come to realize that quadratic programming might yield better performance amidst turbulent markets [3].

Portfolio optimization also depends upon the economic cycles. Graham and Harvey (2001) conducted a study that proved investors' preferences for risk and performance vary during different phases of an economic cycle, namely expansion, recession, and recovery [4]. While expansions make them take more risks, during recession times, conservative strategies are what investors move toward, which is a major, invaluable insight into portfolio management.

Where mean variance remains applicable today to many investors, its use in managing risk for more complicated market conditions is somewhat narrowed. Advanced methods of quadratic programming, CVaR, VaR, and the lower half variance allow investors who apply them to understand better the volatility of their target markets and to realize higher risk-adjusted returns over different economic cycles. This paper will compare the performance of quadratic programming optimization against the mean variance model for different states of the economy and help investors make better asset allocation decisions.

3. Research Methodology

This paper will compare the difference in performance between a portfolio based on quadratic programming optimization and a mean-variance model in different time periods. Steps of the experimental methodology will be

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explained in greater detail in the respective sections below: data collection and processing, experimental design, and results evaluation.

3.1 Data Collection and Processing

3.1.1 Data Source

The data for this study comes from Yahoo Finance, which covers the daily return data of many representative stocks worldwide, laying a solid data foundation for the empirical analysis. We initially collected daily return data from 500 stocks and screened them based on the correlation between these stocks. To ensure the diversification and independence of the selected stocks, we selected stocks with correlation coefficients below 0.3 among them to reduce the impact of inter-asset correlation on the optimization results.

3.1.2 Stock Selection

By calculating the correlation matrix of 500 stocks and screening stocks with correlation coefficients less than 0.3, the following five stocks were finally selected for the portfolio optimization study:

(1) Apple Inc. (AAPL)
(2) Alibaba Group (BABA)
(3) Uber Technologies Inc. (UBER)
(4) Lucid Group, Inc. (LCID)
(5) United Airlines Holdings, Inc. (UAL)

Stock Code	Average Rate of Return	Fluctuation Rate
AAPL	0.12	0.30
BABA	0.08	0.25
UBER	0.10	0.35
LCID	0.06	0.40
UAL	0.09	0.28

Table 1 Mean Returns and Fluctuation Rate for Five Stocks

3.1.3 Data processing

The daily return of each stock is calculated based on its closing price with the following formula:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad (1)$$

where R_t is the return on day t, P_t is the closing price of the day, P_{t-1} is the closing price of the previous day. Then, we calculated the return mean and covariance matrix of each stock for the subsequent portfolio optimization model.

3.2 Experimental design

The core of this study is to compare the performance of the mean-variance model and the quadratic programming model under different periods, and the specific design steps are as follows:

3.2.1 Classification of economic cycles

Based on macroeconomic data and market trends, we divide the sample period (2013 to 2023) into four economic cycles:

3.2.1 .1. 2013-2015: Economic Expansion Period

During this period, the global economy is gradually coming out of the shadow of the 2008 financial crisis, the market becomes more active, and the stock market continues to rise. The reason for choosing this period as the expansion period is that the global GDP showed a continuous growth trend, corporate profits improved significantly, and the easing policies adopted by central banks, especially the economic boost from the United States and China, further enhanced the global economic vitality.

3.2.1 .2. 2016-2018: Period of economic stabilization

During these years, the global economy entered a relatively stable state. Although there were minor fluctuations, the overall economy maintained relatively stable growth. This phase was chosen as a plateau since the growth rates of major global economies have converged, the unemployment rate is at a low level, and the inflation rate has remained stable, showing that the market has entered a mature and stable phase.

3.2.1 .3. 2019-2021: Recession and recovery period

Due to the impact of the New Crown Epidemic, the global economy experienced a severe recession during this period, especially in 2020 when the economic downturn was severe. However, the economy begins to recover gradually in 2021 as vaccination becomes widespread and countries implement large-scale economic stimulus policies. Therefore, defining this period as a recession and recovery period reflects the far-reaching impact of the epidemic on the economy and the subsequent recovery process.

3.2.1 .4. 2021-2023: Economic Recovery Period

During this period, the global economy gradually shakes off the impact of the epidemic and begins to gradually return to the right track, with investment and consumer confidence restored. The reason for choosing this period as the recovery period is that the economic policies of various countries gradually return to normal, the global supply chain improves, the profitability of enterprises strengthens, the global trade becomes active again, and the indicators such as GDP, employment rate and consumption level show that the economic recovery is obvious.

3.2.2 Portfolio Optimization Model

3.2.2 .1 Mean-Variance Model

The mean-variance model optimizes the portfolio by minimizing the risk (variance) of the portfolio and maximizing the expected return. Its optimization problem is expressed as

$$Minimize\ \sigma_p^2 = w^T \sum w \quad (2)$$

where σ_p^2 represents the variance of the portfolio, w is a vector of asset weights, and $\sum$ is the covariance matrix of asset returns.

3.2.2 .2. Quadratic planning models

The quadratic programming model introduces more sophisticated risk measurement tools such as Value at Risk (VaR), Conditional Value at Risk (CVaR) and Lower Semivariance. Its optimization objective is:

$$Minimize\ f(w) = \lambda_1\sigma_p^2 + \lambda_2 Semivariance + \lambda_3 CVaR_\alpha \quad (3)$$

$$Subject\ to \sum_{i=1}^{n} w_i = 1, 0 \leqslant w_i \leqslant 1 \quad (4)$$

where λ_1, λ_2, λ_3 are the weight parameter of a risk measure and w is the weight vector of the portfolio.

With the help of the Python library cvxpy, the optimization process is solved to get controlled performance of the portfolio under different risk measures. By this, the approach aims for the portfolio to be more resilient regarding the risks from market volatility.

3.3 Evaluation of results

3.3.1 Comparison of valid boundary plots

This paper evaluates the inefficient frontier performance over time for the mean-variance and quadratic programming models by comparing their efficient frontiers. These plots show the optimal returns that each model can achieve at various levels of risk, thus showing which of the models is performing better under differing economic conditions.

3.3.2 Comparison of Sharpe Ratio

It is hence obvious that the Sharpe ratio yields a relation of a portfolio's excess return over the risk-free rate to volatility. The Sharpe ratios of the two models are computed for each economic cycle in this paper as follows:

$$Sharpe\ Ratio = \frac{E(R) - R_f}{\sigma} \quad (5)$$

where $E(R)$ is the expected return of the portfolio, R_f is the risk-free rate, σ is the volatility of the portfolio.

This will, in turn, help us to establish which of the two models has a better risk-return ratio for different economic cycles by comparing their Sharpe ratios.

4. Empirical Analysis

4.1 Economic expansion period (2013-2015)

During the period of economic expansion, the market is in a high-growth zone, and investors use a high preference for risky assets. Currently, both the optimized quadratic programming model and mean-variance model show better return characteristics.

4.1.1 Comparison of effective boundaries

According to Figure 1, the effective boundaries of both models at this stage show the characteristics of high return and low risk. The effective boundary of the quadratic programming model is significantly better than that of the mean-variance model, especially in the high-risk interval, the optimization model can provide higher returns.

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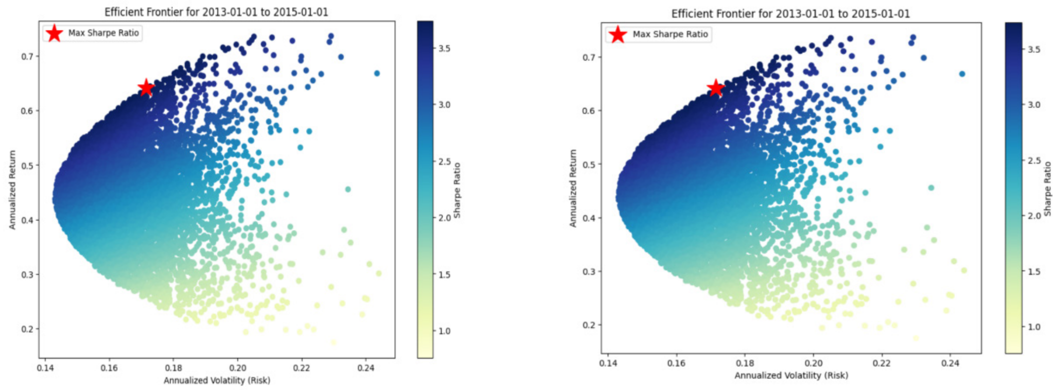

Fig. 1 Effective boundary image of the mean-variance model (left) and effective boundary image of the quadratic programming model (right)

4.1.2 Sharpe ratio analysis

As can be seen from Table 2, the Sharpe ratio of the quadratic programming model is higher than that of the mean-variance model. Specifically, the Sharpe ratio of the quadratic programming model is 1.47, while that of the mean-variance model is 1.24, which shows that the quadratic programming model has a stronger risk-adjusted return advantage under high-risk conditions.

Table 2 Sharpe ratios for mean-variance model and quadratic programming model

Economic cycles	Mean-variance model	Quadratic programming model
Economic expansion period	1.24	1.47

4.2 Period of economic stabilization (2016-2018)

During the economic stabilization period, the market is less volatile and the performance of the two models is closer. Investors are less sensitive to risk and the need for risk control is not as strong as in the recessionary period.

4.2.1 Comparison of effective boundaries

Figure 2 illustrates the effective boundaries for this period. The effective boundaries of the quadratic programming model and the mean-variance model are close to each other, but the quadratic programming model still shows a better risk-return ratio in the high-return interval.

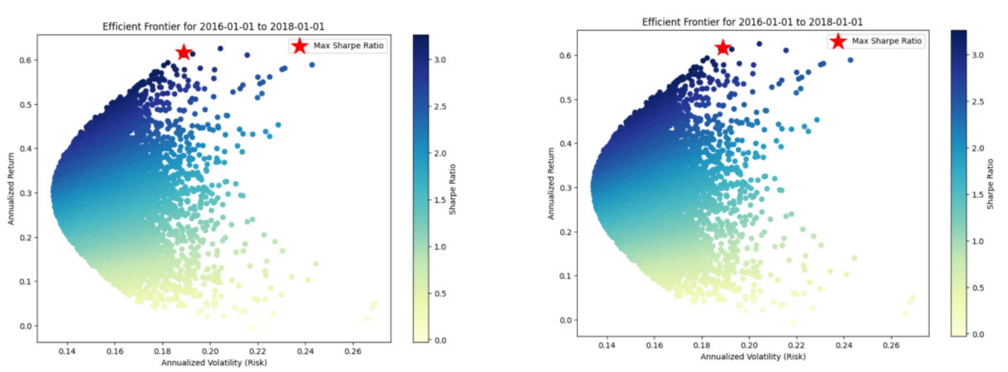

Fig. 2 Effective boundary image of the mean-variance model (left) and effective boundary image of the quadratic programming model (right)

4.2.2 Sharpe ratio analysis

As Table 3 shows, in terms of Sharpe ratio, there is not much difference between the two models. The Sharpe ratio of the quadratic programming model is 0.91, which is slightly higher than that of the mean-variance model, which is 0.70. The quadratic programming model manages to maintain higher returns despite the lower risk.

Table 3 Sharpe ratios for mean-variance model and quadratic programming model

Economic cycles	Mean-variance model	Quadratic programming model
Economic stabilization period	0.70	0.91

4.3 Period of economic recession (2019-2021)

The global recession and increased market volatility due to the arrival of the New Crown Epidemic, at this time, puts higher demands on risk management. The quadratic planning model introduces advanced risk control tools such as VaR and CVaR, which can better cope with extreme market risks.

4.3.1 Effective Boundary Comparison

Figure 3 shows the effective boundaries during the recession period. The effective boundary of the quadratic programming model outperforms the mean-variance model in terms of risk control, especially in the low-risk region, where the optimized portfolio can provide more stable returns.

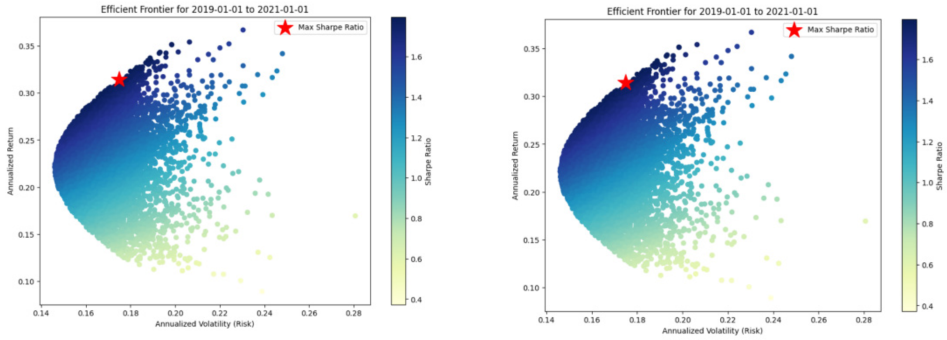

Fig. 3 Effective boundary image of the mean-variance model (left) and effective boundary image of the quadratic programming model (right)

4.3.2 Sharpe ratio analysis

In Table 4, the Sharpe ratio of the quadratic programming model in the recession period is 1.68, which is significantly higher than that of the mean-variance model at 1.58. This indicates that the quadratic programming model performs better in controlling extreme risks during periods of high volatility and can provide better risk-adjusted returns to investors.

Table 4 Sharpe ratios for mean-variance model and quadratic programming model

Economic cycles	Mean-variance model	Quadratic programming model
Economic recession period	1.58	1.68

4.4 Economic recovery period (2021-2023)

As the global economy gradually recovers from the epidemic, market performance gradually picks up. Investors' risk appetite picks up, and both models perform more robustly during the recovery period.

4.4.1 Comparison of Effective Boundaries

During the economic recovery period (Figure 4), the effective boundaries of the quadratic programming model and the mean-variance model are very close to each other, and both of them show good return characteristics in the low and medium risk ranges. In the high-risk range, the quadratic programming model still slightly outperforms the mean-variance model.

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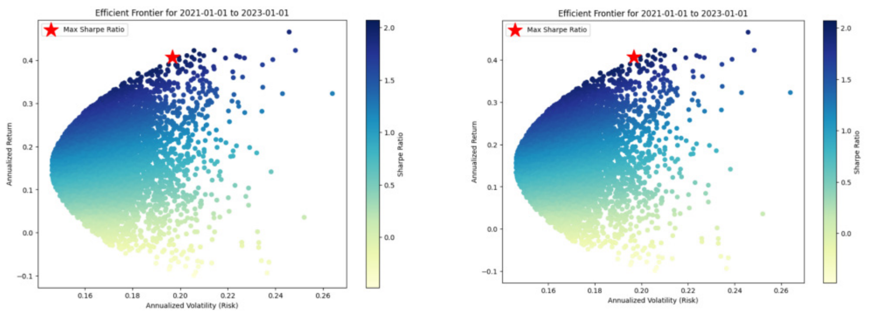

Fig. 4 Effective boundary image of the mean-variance model (left) and effective boundary image of the quadratic programming model (right)

4.4.2 Sharpe Ratio Analysis

As Table 5 shows, the difference between the Sharpe ratios of the two models is relatively small, with the Sharpe ratio of the quadratic programming model being 1.46, while that of the mean-variance model is 1.42, showing that the quadratic programming model is still slightly more able to cope with market volatility.

Table 5 Sharpe ratios for mean-variance model and quadratic programming model

Economic cycles	Mean-variance model	Quadratic programming model
Economic recovery period	1.42	1.46

5. Summary

This paper analyzes the performance differences between the quadratic programming-based portfolio optimization model and the traditional Markowitz mean-variance model in different economic cycles by comparing the two models. The results show that the quadratic programming model can control risk more effectively in most cases, and especially exhibits higher risk-adjusted returns in economic cycles with high market volatility. The following is an in-depth summary of the main results and a financial perspective.

During periods of economic expansion, risky assets such as equities outperform as investors' risk appetite increases in line with macroeconomic growth and strong capital market performance. The quadratic programming model exhibits higher Sharpe ratios and better efficient bounds by optimizing return and risk and is more flexible in allocating risky assets. In contrast, the mean-variance model, while also providing stable returns, is limited in its ability to manage extreme return volatility. During expansionary periods, the quadratic programming model allows investors to better hedge extreme risks while pursuing higher returns through precise VaR (Value at Risk) and CVaR (Conditional Value at Risk) controls.

During periods of economic stabilization, market volatility declines and overall risk is low. During this phase, the mean-variance model performs close to the quadratic programming model due to its simplicity and stability. Nonetheless, the quadratic programming model still has a slight advantage in its ability to control risk, especially in the high-yield range. For investors seeking long-term holdings during the stabilization period, the quadratic programming model provides additional protection against downside risk management through the introduction of the lower half of the variance. The mean-variance model has a slightly lower Sharpe ratio, indicating a slightly higher volatility of returns at extreme risk.

During a recession, global markets experience major shocks, market liquidity tightens, prices of risky assets fall sharply, and systemic risk rises. The quadratic programming model can provide significant downside protection in times of severe market volatility by introducing more sophisticated risk measurement tools and is particularly superior in dealing with tail risks such as stock market crashes. In contrast, mean-variance models struggle to capture extreme asymmetric risks in the market due to their reliance on the variance of the covariance matrix for risk assessment. In this environment, the quadratic programming model not only improves the portfolio's resilience to downturns, but its Sharpe ratio is also significantly higher than that of the mean-variance model during recessions, demonstrating its superiority in dealing with market volatility.

During the economic recovery period, the market gradually recovers from the recession, and corporate earnings and investor confidence gradually rebound. At this time, the market demand for risky assets increases and returns and risks are balanced again. At this stage, the effective boundaries of the two models are close, but the quadratic programming model still shows better risk-adjusted returns, especially in the allocation of high-volatility assets is more flexible and can better capture the market opportunities brought by the recovery period. Meanwhile, by reducing the probability of tail risk, the quadratic programming model can maintain portfolio robustness over a longer period, while the mean-variance model is more suitable for short-term investors or less volatile market environments.

6. Financial Applications and Practical Implications

The findings of this paper provide practical references for investors to choose optimization models in different economic cycles. During expansions and recoveries, investors face more opportunities, and quadratic programming models can provide higher potential returns and effectively manage extreme market volatility. When the market is performing well, the quadratic programming model utilizes CVaR to capture high-return opportunities while avoiding losses due to sudden market corrections through strict tail risk control.

During recession, the market environment deteriorates, investors' risk appetite decreases, and the safety of assets becomes a primary concern. The quadratic programming model has significant risk tolerance by introducing sophisticated risk control mechanisms to help investors reduce losses under extreme market conditions. The mean-variance model is still efficient when the economy is less volatile and is suitable for short-term operations or conservative investors. However, it does not perform as well as the quadratic programming model in terms of risk-adjusted returns when facing extreme market conditions.

Overall, the quadratic programming optimization model performs more flexibly and robustly in terms of risk control and return adjustment and is more capable of providing stable returns to investors, especially during high volatility phases such as recessions and recoveries. As the complexity of the financial market environment increases, future research could further explore the enhancement of the optimization strategy by introducing other advanced risk measurement tools (e.g., CVaR combined with dynamic risk premium models), especially under different asset classes and market conditions. For portfolio management practitioners, the judicious application of quadratic programming optimization models can help address market challenges in different economic cycles and enhance long-term returns and risk management capabilities.

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