

# APPLICATION OF THE MARKOWITZ MODEL AND INDEX MODEL IN REAL STOCK MARKETS

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## Abstract

The complete Markowitz Model and the Index Model have a wide range of applications for portfolios in the stock market, and in this paper, we want to apply these two models in a realistic stock market. In this paper, ten stocks belonging to different industries and the S&P 500 equity index are selected as a portfolio, and the 1-month Fed Funds rate is chosen as the risk-free rate. Through proper data aggregation and processing, we calculate the weights of each stock in the portfolio in the case of minimal portfolio variance and maximal Sharpe ratio under five additional constraints using the complete Markowitz Model and the Index Model, respectively. Finally, we can calculate the minimal variance frontier, efficient frontier, inefficient frontier, and capital allocation line under each constraint, plotted using the Solver Table in Excel.

**Keywords:** Markowitz Model; Index Model; portfolio; additional constraint; minimal variance frontier

## 1 Introduction

### 1.1 Research background

The securities market is a market full of diversity and randomness. Therefore, as an investor, obtaining additional returns by purchasing securities is the primary purpose of entering the securities market. However, in the securities market, as in many areas, high returns mean high risk. However, most investors are risk-averse, wanting higher returns but unwilling to take higher risks. To cope with this problem, Markowitz proposed the complete Markowitz Model, which means that portfolio diversification can reduce risk without significantly affecting portfolio returns. At the same time, to address Markowitz's excessive data input in the case of large samples, a new portfolio model, the Index Model, was proposed, which significantly reduces the amount of data needed for the calculation. The primary purpose of both models is to calculate indicators such as the return and risk of a portfolio by the weights of each stock in the portfolio. Therefore, in the reality of investing, we want to construct a specific portfolio based on the estimation of return and risk.

### 1.2 Our work

Therefore, we want to obtain data in fundamental stock markets from 2001 to 2021 and perform appropriate data processing to satisfy the prerequisites for applying both models. At the same time, we also introduce five additional constraints, which are a series of computational restrictions imposed on the weights of each stock to match the real stock market closely. By applying both models, we can calculate the maximum return or minimize risk for

a given portfolio and plot the minimal variance frontier with the efficient investment frontier to facilitate investors' investment choices. We also compare the similarities and differences between the complete Markowitz Model and the Index Model under each additional constraint and try to explain the reasons behind them.

## 2 Notations

The necessary mathematical notations used in this paper are listed in Table 1.

**Table 1: Notations used in this paper.**

Symbol	Description
$w$	Weight of each stock
$\mu$	The average return of each stock
$\sigma$	Standard deviation
$\beta$	Sensitivity coefficient
$\alpha$	Expected excess return
$e$	Residual
$\rho$	Cross-correlation coefficient
$r$	Return rate
$k$	Stock index

## 3 Mathematical principles of models

### 3.1 The complete Markowitz Model

In the full Markowitz Model, we achieve diversification and risk reduction by assigning a certain weight to each stock. The return of the portfolio can be written as

$$r_P = w_1\mu_1 + w_2\mu_2 + \cdots + w_n\mu_n = \sum_{i=1}^n w_i\mu_i$$

Where:

- $w_1 + w_2 + \cdots + w_n = 1$

- I represent the i-th stock in the portfolio.

For ease of reading, we can express the mathematical formula in matrix form.

$$\vec{w} = (w_1, w_2, \dots, w_n)$$

$$\vec{\mu} = (\mu_1, \mu_2, \dots, \mu_n)$$

$$r_p = \vec{w}\vec{\mu}^T$$

When calculating the risk of a portfolio, we use the standard deviation of the portfolio as a measure. We continue to use a calculation that sums the variance of each stock based on the weight of the stock in the portfolio. However, because of the correlation between stocks, the variance calculation also considers the covariance coefficient between the two stocks. Therefore, the mathematical formula can be written as

$$\sigma_p = \sqrt{\sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_i \sigma_j \rho_{ij}}$$

Where:

- i and j represent the i-th and j-th stock in the portfolio, respectively.

For ease of reading, we can express the mathematical formula in matrix form.

We first form an auxiliary vector.

$$\vec{v} = (w_1 \mu_1, w_2 \mu_2, \dots, w_n \mu_n)$$

Then we form the matrix of the instruments' cross-correlation coefficients.

$$P = \begin{pmatrix} \rho_{11} & \rho_{12} & \dots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \dots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \dots & \rho_{nn} \end{pmatrix}$$

Where:

- The elements on the diagonal are all one because each stock has a correlation coefficient of 1 with itself.

Thus the standard deviation of the portfolio can be expressed as

$$\sigma_p = \sqrt{\vec{v}P\vec{v}^T}$$

### 3.2 the Index Model

The full Markowitz Model is very well developed but has several drawbacks. First, the Markowitz model must evaluate each stock's return, standard deviation, and the

correlation coefficient between stocks. When the number of stocks in a portfolio is large, the data to analyze increases significantly. Besides, some sets of correlation coefficients can be mutually inconsistent. Therefore, a new model, the index model, was born.

In the Index Model, the return formula for each stock is

$$r_i - r_f = \alpha_i + \beta_i (r_m - r_f) + e_i$$

Where:

- $r_{rf}$  is the return rate of a risk-free asset.
- $r_{rm}$  is the return rate of the market.
- I represent the i-th stock in the portfolio.

Like the full Markowitz Model, the return of the portfolio can still be expressed as

$$r_p = \vec{w}\vec{\mu}^T$$

Because the index model no longer considers the correlation coefficient between stocks, the calculation of the standard deviation of the portfolio differs significantly from the Markowitz model. We first calculate the beta of the portfolio.

$$\vec{\beta} = (\beta_1, \beta_2, \dots, \beta_n)$$

$$\beta_p = \vec{w}\vec{\beta}^T$$

Where:

- A least squares regression is performed on the stock data to calculate a linear function, and the slope of the process is the  $\beta$ .

Then we can derive the standard deviation of the portfolio.

$$\sigma_p = \sqrt{(\sigma_m \beta_p)^2 + \sum_{i=1}^n w_i^2 \sigma^2(e_i)}$$

Where:

- $\sigma_m$  is the standard deviation of the common macroeconomic factor.
- I represent the i-th stock in the portfolio.

## 4 Acquisition, processing, and presentation of data

### 4.1 Data source

We selected stocks of ten companies from different industries from 2001 to 2021. We present a table plotting the stocks' position in the portfolio, their abbreviations, and the drive to which they belong by full name.

**Table 2: Stocks used in this paper.**

Index	Abbreviation	Full Name	Sector
1	SPX	S&P 500 Index	
2	AMZN	Amazon.com, Inc.	Consumer Cyclical
3	AAPL	Apple Inc.	Technology
4	CTXS	Citrix Systems, Inc.	Technology
5	JPM	JPMorgan Chase & Co.	Financial Services
6	BRK/A	Berkshire Hathaway Inc.	Financial Services

Index	Abbreviation	Full Name	Sector
7	PGR	The Progressive Corporation	Financial Services
8	UPS	United Parcel Service, Inc.	Industrials
9	FDX	FedEx Corporation	Industrials
10	JBHT	J.B. Hunt Transport Services, Inc.	Industrials
11	LSTR	Landstar System, Inc.	Industrials

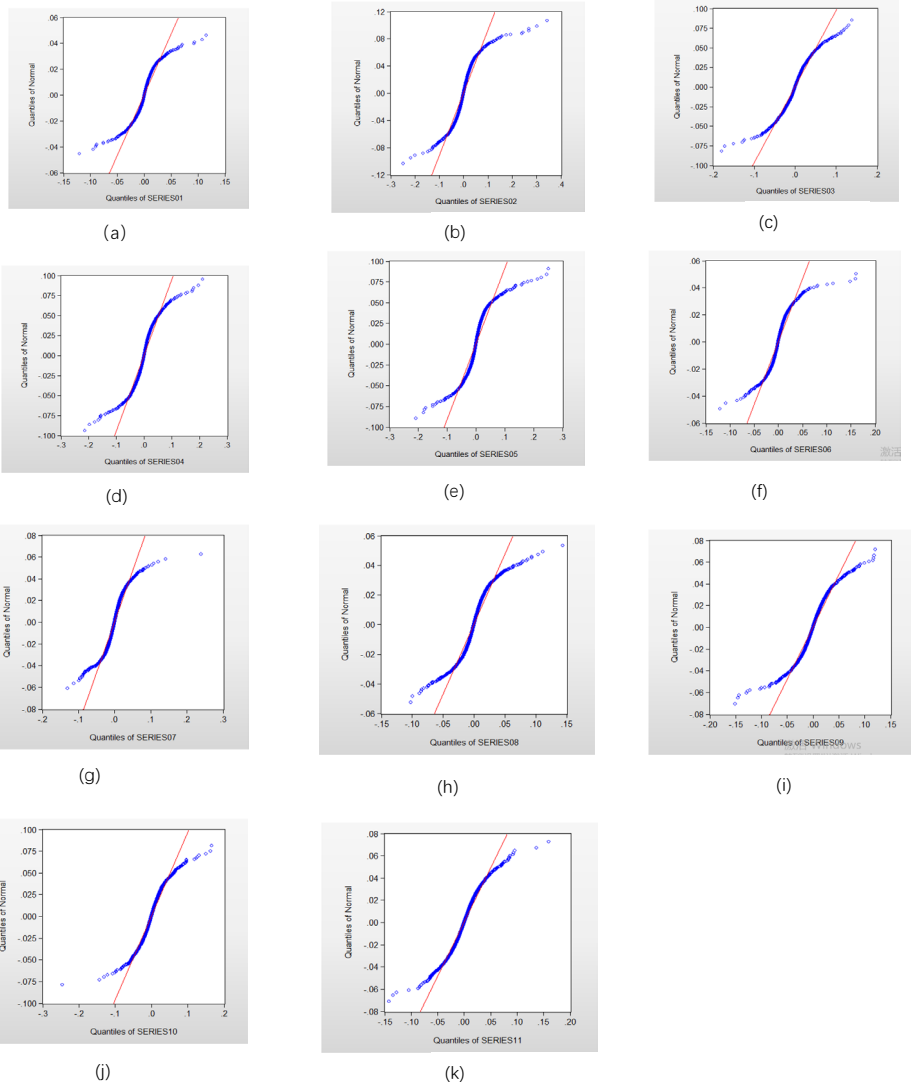
#### 4.2 daily return rate and monthly return rate

We can access the index of each stock for each trading day on the website. We need a formula to convert the indices into return rates.

$$r_{i,d} = \frac{k_{i,d}}{k_{i,d-1}} - 1$$

Where:

- $i$  represent the  $i$ -th stock in the portfolio.
  - $t$  represents the return of the stock on day  $d$ .
- Using the above mathematical formula, we can get the return of each stock for each trading day. Now we need to test one thing. The full Markowitz and Index Model require that the stock's returns satisfy a normal distribution. Therefore, we use Eviews to verify whether the daily return rates of each stock satisfy a normal distribution.



**Figure 1 Quantile-Quantile figures(daily return): (a)SPX, (b)AMZN, (c)AAPL, (d)CTXS, (e) JPM, (f)BRK/A, (g)PGR, (h)UPS, (i)FDX, (j)JBHT, (k)LSTR**

The principle of the Quantile-Quantile figure is that the closer the data points are to the red line, the closer the

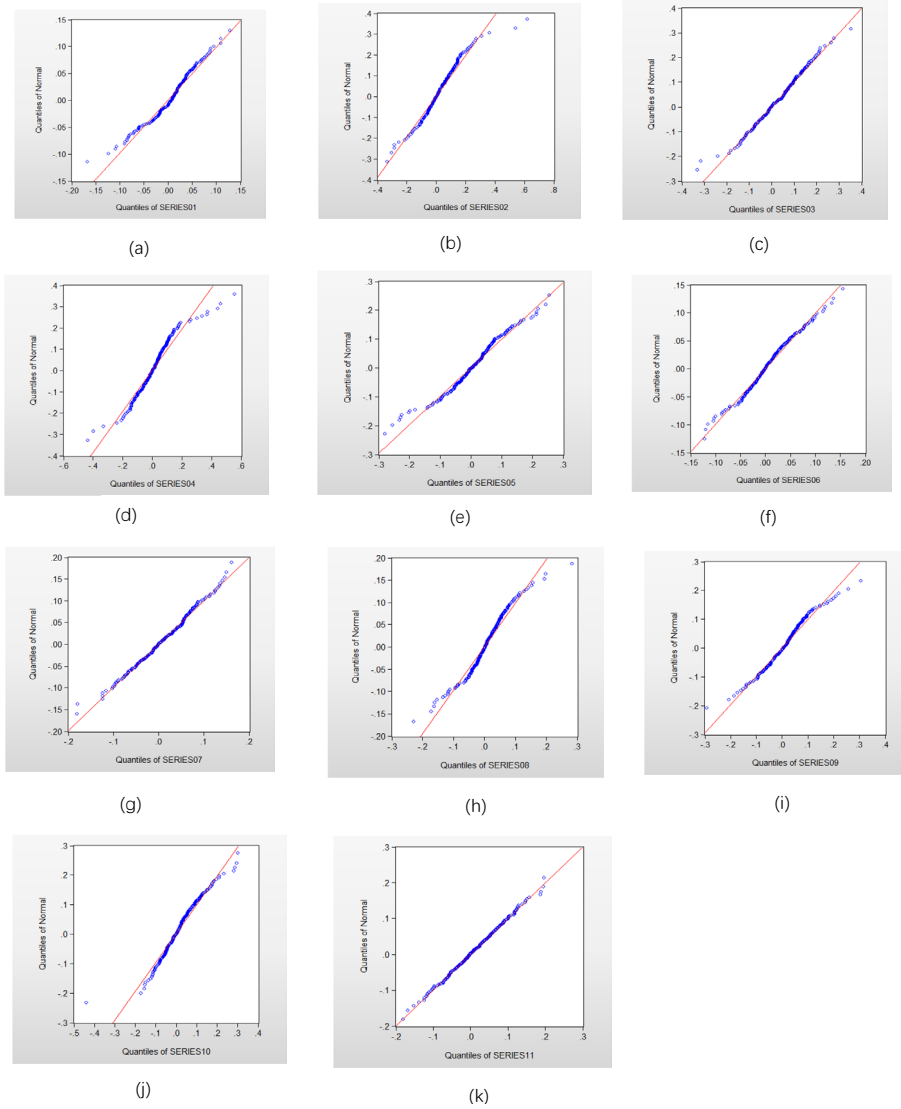
data are to a normal distribution. As you can see from the eleven Quantile-Quantile figures above, the data points at both ends are relatively far from the red line, indicating that the daily return rates are not very close to a normal distribution. This also means that we need to process the data to make it closer to a normal distribution. Therefore, according to the central limit theorem, we transform the daily return into a monthly return. The conversion formula is as follows.

$$\begin{aligned}\Delta k_{i,m} &= \Delta k_{i,1} + \Delta k_{i,2} + \dots + \Delta k_{i,t-1} \\ &= (k_{i,2} - k_{i,1}) + (k_{i,3} - k_{i,2}) + \dots + (k_{i,t} - k_{i,t-1}) \\ &= k_{i,t} - k_{i,1}\end{aligned}$$

$$r_{i,m} = \frac{\Delta k_{i,m}}{k_{i,1}} = \frac{k_{i,t}}{k_{i,1}} - 1$$

Where:

- $i$  represent the  $i$ -th stock in the portfolio.
  - $r_m$  represents the return of the stock at month  $m$ .
- Here we sum the difference between the stock indices every two days, and from that, we calculate the monthly return of the stock. This treatment will bring the data distribution closer to a normal distribution. Then, we still use Eviews to verify whether the monthly return rates of each stock satisfy a normal distribution.



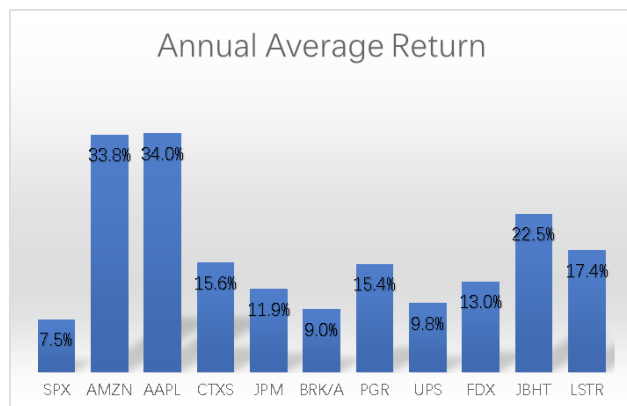
**Figure 2 Quantile-Quantile figures(monthly return): (a)SPX, (b)AMZN, (c)AAPL, (d)CTXS, (e)JPM, (f)BRK/A, (g)PGR, (h)UPS, (i)FDX, (j)JBHT, (k)LSTR**

As can be seen, the Quantile-Quantile figure of monthly returns for each stock is closer to the red line than the daily returns. This means the monthly returns can be approximated to obey a normal distribution. Therefore, all

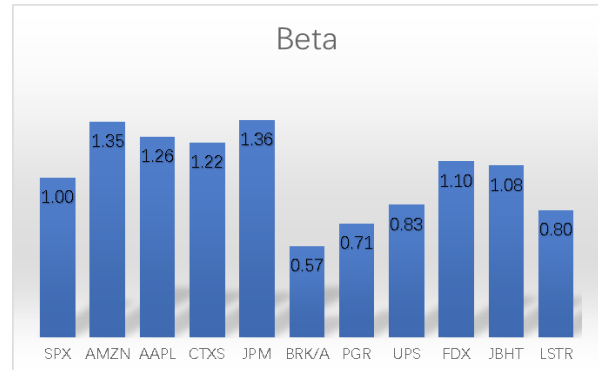
subsequent operations will be done based on the monthly returns.

## 4.3 Calculation of other indicators

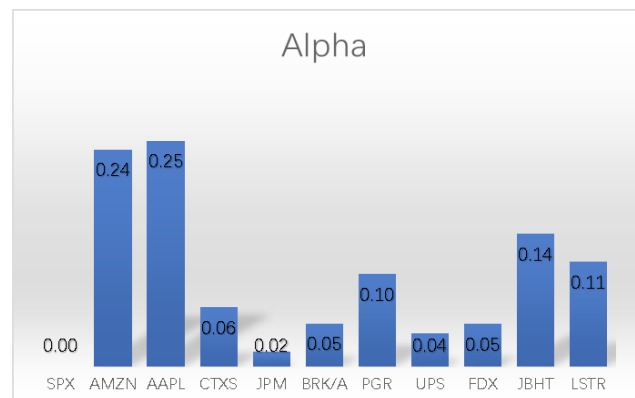
We have calculated the monthly return of the stock above. First, we need to subtract the risk-free rate to get the excess return rate. Then we calculate each stock's average return and standard deviation based on the excess return rate and use least squares estimation to calculate each stock's beta and alpha values. Finally, we also need to calculate the standard deviation of the residuals obtained from the regression.



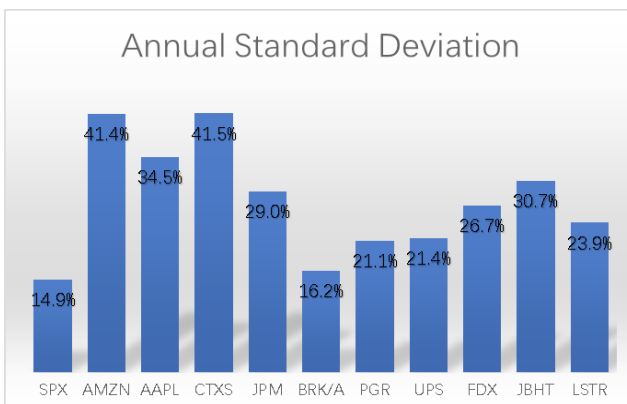
**Figure 3 shows the annual average return of each stock**



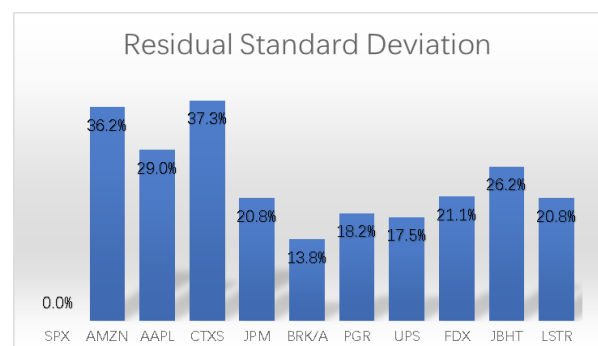
**Figure 5 shows the Beta of each stock**



**Figure 6 The Alpha of each stock**



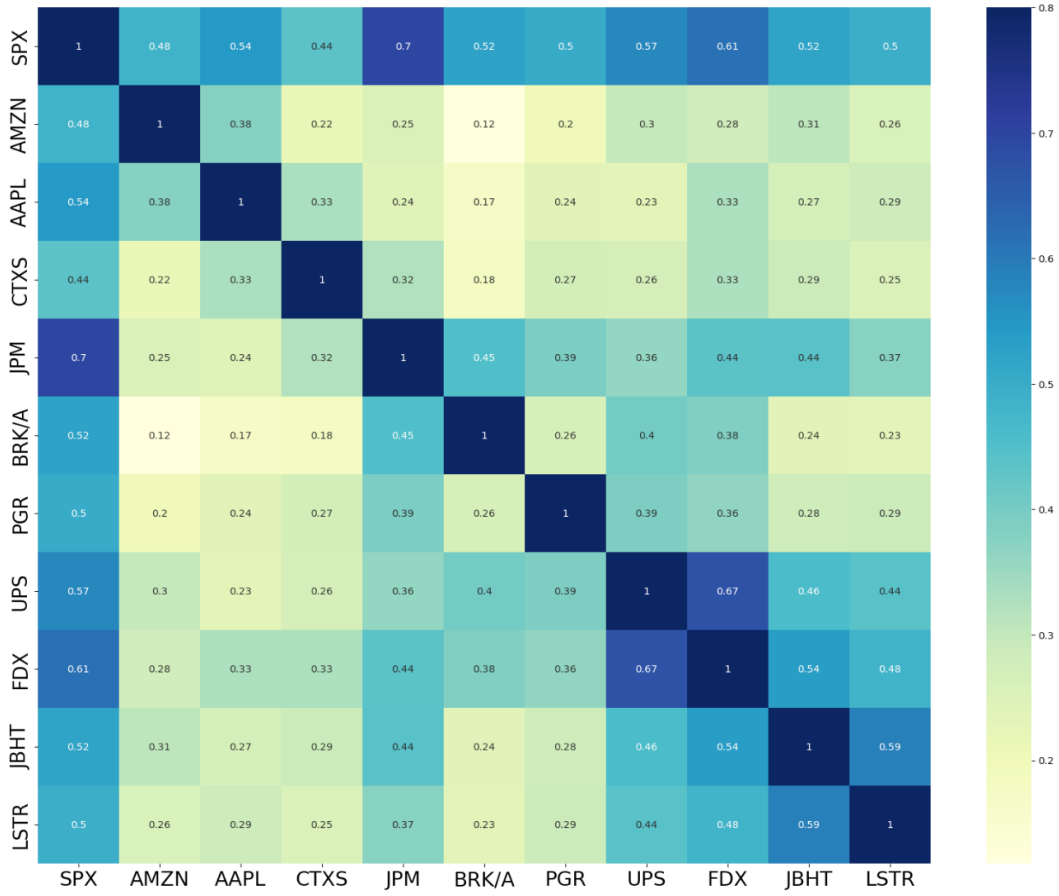
**Figure 4 shows the Annual Standard Deviation of each stock**



**Figure 7 the Residual Standard Deviation of each stock**

## 4.4 Calculation and visual presentation of the correlation coefficient

In the above, we obtained the monthly return of the stock with standard deviation. Here we can calculate the covariance between the two stocks from the monthly returns and the correlation coefficient between them. We use Python to plot the correlation coefficient heatmap of the stocks.



**Figure 8 The heatmap of the correlation coefficient**

As we can see from the graph, the correlation coefficient between most of the stocks is still relatively small. However, the correlation coefficients between SPX and all other stocks are relatively large, and the correlation coefficients among four stocks, UPS, FDX, JBHT, and LSTR, are also relatively large. These four companies all belong to the logistics and transportation industry, which also shows a greater influence between different companies in the transportation industry.

## 5 Five additional constraints

After the basic data processing and indicator calculation, we need to consider additional constraints on the model to match the realistic stock market conditions better. We will perform five additional restrictions, including four on stock weights and a blank control group.

### 5.1 Constraint 1

This additional optimization constraint is designed to simulate Regulation T by FINRA, which allows broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity:

$$\sum_{i=1}^n |w_i| \leq 2$$

### 5.2 Constraint 2

This additional optimization constraint is designed to simulate some arbitrary “box” constraints on weights, which the client may provide:

$$|w_i| \leq 1, \text{ for } \forall i$$

### 5.3 Constraint 3

A “free” problem, without any additional optimization constraints, to illustrate how the area of permissible portfolios in general and the efficient frontier in particular look like if you have no restrictions.

### 5.4 Constraint 4

This additional optimization constraint is designed to simulate the typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund cannot have any short positions.

$$w_i \geq 0, \text{ for } \forall i$$

## 5.5 Constraint 5

Lastly, we would like to see if the inclusion of the broad index into our portfolio has a positive or negative effect; for that, we would like to consider an additional optimization constraint:

$$w_1 = 0$$

## 6 Minimal Risk Portfolio and Maximal Sharpe Ratio

After setting the additional constraints, we want to calculate the portfolio's minimal risk and maximal Sharpe Ratio under different models with different additional constraints. We will use a linear programming approach

for this calculation.

### 6.1 Minimal Risk Portfolio

The linear programming formula for calculating the minimal standard deviation of a portfolio can be written as

$$\begin{cases} \sigma_P(\vec{w}) \text{ @min} \\ \text{constrain } i = 1, 2, 3, 4, 5 \end{cases}$$

We apply the full Markowitz Model and the Index Model to calculate the minimal standard deviation of the portfolio under each additional constraint and the corresponding weights of each stock. We plot the results in the following table.

**Table 3: Minimal Risk Portfolio under the full Markowitz Model.**

Constraint	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR	StDev
1	72.24%	-2.35%	-3.85%	-1.04%	-18.47%	36.21%	13.91%	3.43%	-10.28%	-0.55%	10.75%	12.24%
2	72.24%	-2.35%	-3.85%	-1.04%	-18.47%	36.21%	13.91%	3.43%	-10.28%	-0.55%	10.75%	12.24%
3	72.24%	-2.35%	-3.85%	-1.04%	-18.47%	36.21%	13.91%	3.43%	-10.28%	-0.55%	10.75%	12.24%
4	38.47%	0.00%	0.00%	0.00%	0.00%	38.49%	14.23%	0.72%	0.00%	0.00%	8.09%	13.09%
5	0.00%	2.45%	4.19%	0.84%	-7.09%	56.31%	23.52%	11.41%	-8.05%	0.60%	15.82%	13.39%

**Table 4: Minimal Risk Portfolio under the Index Model.**

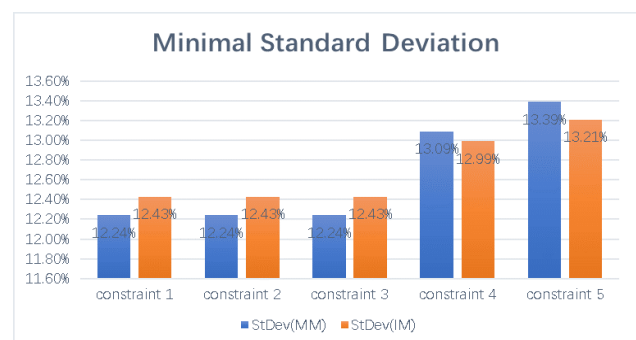
Constraint	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR	StDev
1	65.79%	-4.14%	-4.74%	-2.44%	-12.87%	34.51%	13.42%	8.56%	-3.61%	-1.72%	7.24%	12.43%
2	65.79%	-4.14%	-4.74%	-2.44%	-12.87%	34.51%	13.42%	8.56%	-3.61%	-1.72%	7.24%	12.43%
3	65.79%	-4.14%	-4.74%	-2.44%	-12.87%	34.51%	13.42%	8.56%	-3.61%	-1.72%	7.24%	12.43%
4	30.46%	0.00%	0.00%	0.00%	0.00%	37.65%	14.64%	9.34%	0.00%	0.00%	7.90%	12.99%
5	0.00%	-1.55%	-0.81%	-0.11%	-5.02%	47.99%	21.65%	17.82%	3.45%	2.82%	13.76%	13.21%

We now compare and present the minimal standard deviations obtained from the two models calculated under different additional constraints.

As can be seen from the figure, the full Markowitz Model is consistent under the first three additional constraints. In comparison, the results under the last two other constraints are larger, which is also for the Index Model. It illustrates the relatively small effect of constraint one and constraint two on the minimal standard deviation of the portfolio, while constraints 4 and 5 increase the minimal standard deviation of the portfolio.

When we compare the computational results of the two models, we find that the minimal standard deviation of the Index Model is more significant than that of the complete Markowitz Model under the first three additional constraints. In contrast, the opposite is valid under the last two additional constraints. The reason may be that the

Index Model adds other variables and assumptions.



**Figure 9 shows the comparison of the minimal standard deviation.**

### 6.2 Maximal Sharpe Ratio

The Sharpe Ratio is a measure of the risk and return of

a portfolio that compares the risk and return of different portfolios and thus helps investors make investment decisions. The Sharpe Ratio is calculated by the Sharpe formula, which is the ratio between investment returns and risk.

The linear programming formula for calculating the maximal Sharpe Ratio of a portfolio can be written as

$$\begin{cases} \frac{r_P(\bar{w})}{\sigma_P(\bar{w})} \text{max} \\ \text{constraint } i = 1, 2, 3, 4, 5 \end{cases}$$

We apply the full Markowitz Model and the Index Model to calculate the maximal Sharpe Ratio of the portfolio under each additional constraint and the corresponding weights of each stock. We plot the results in the following table.

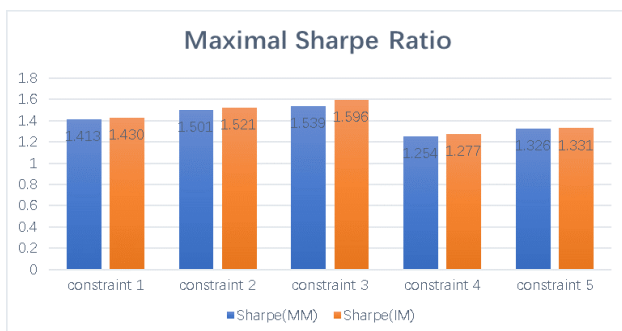
**Table 5: Maximal Sharpe Ratio under the full Markowitz Model.**

Constraint	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR	Sharpe
1	-48.25%	16.40%	30.02%	-0.10%	-0.09%	41.31%	32.96%	-0.02%	-1.46%	12.50%	16.73%	1.413
2	-100.00%	22.33%	39.76%	-1.20%	-0.50%	62.48%	46.01%	-3.11%	-10.56%	20.88%	23.91%	1.501
3	-237.52%	37.06%	65.39%	0.42%	17.30%	91.59%	68.19%	1.27%	-8.69%	30.97%	34.02%	1.539
4	0.00%	12.96%	25.21%	0.00%	0.00%	19.26%	22.72%	0.00%	0.00%	8.81%	11.04%	1.254
5	0.00%	14.65%	26.73%	-3.44%	-15.54%	36.35%	31.99%	-12.17%	-13.21%	17.83%	16.81%	1.326

**Table 6: Maximal Sharpe Ratio under the Index Model.**

Constraint	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR	Sharpe
1	-47.30%	17.84%	30.52%	0.50%	-2.68%	22.44%	31.53%	1.17%	0.11%	19.20%	26.66%	1.430
2	-100.00%	21.85%	36.62%	3.09%	-8.79%	33.32%	39.98%	9.52%	5.62%	24.97%	33.83%	1.521
3	-329.78%	43.02%	69.96%	11.02%	8.83%	58.47%	72.35%	27.91%	24.94%	50.34%	62.93%	1.596
4	0.00%	15.46%	27.12%	0.00%	0.00%	3.26%	21.36%	0.00%	0.00%	13.98%	18.82%	1.277
5	0.00%	18.56%	31.69%	-0.47%	-21.23%	11.67%	27.66%	-5.30%	-5.40%	18.57%	24.25%	1.331

We now compare and present the maximal Sharpe Ratio from the two models calculated under different additional constraints.



**Figure 10 shows the comparison of the maximal Sharpe Ratio.**

The figure shows that the Sharpe ratio is maximum under the full Markowitz Model and the Index Models with constraint 3. It shows that the additional constraint causes the maximal Sharpe Ratio to decrease. Meanwhile, the maximal Sharpe Ratio calculated by the Index Model is

more significant than that calculated by the full Markowitz Model under each additional constraint, but the increase is minimal.

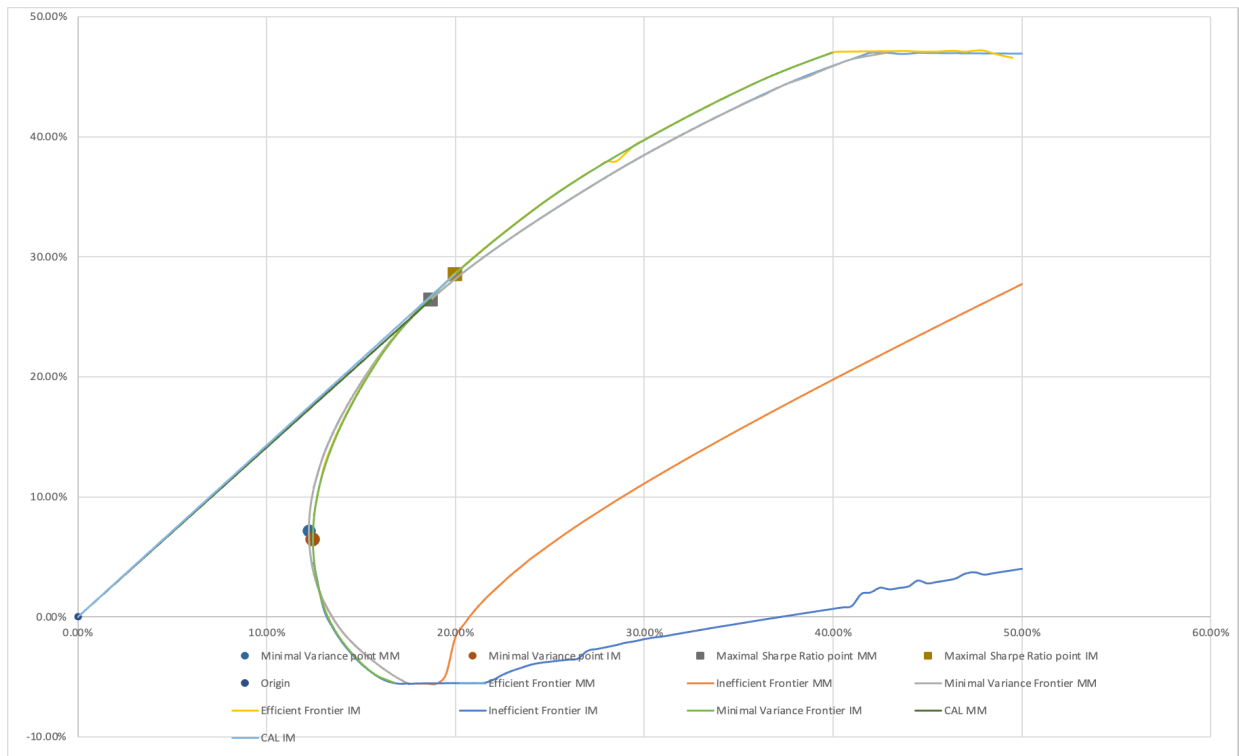
## 7 Frontier and Capital Allocation Line

After finding the minimal standard deviation and the maximal Sharpe Ratio for each model under each additional constraint, we also want to find the maximal return of the portfolio corresponding to the different standard deviations and plot them as curves, which are efficient frontier, inefficient frontier, and minimal variance frontier. Also, we want to find a straight-line tangent to this curve past the origin, which is the capital allocation line. Finally, we want to compare the curve with the straight line under different models with additional constraints. We use the Solver Table in Excel to complete the above work.

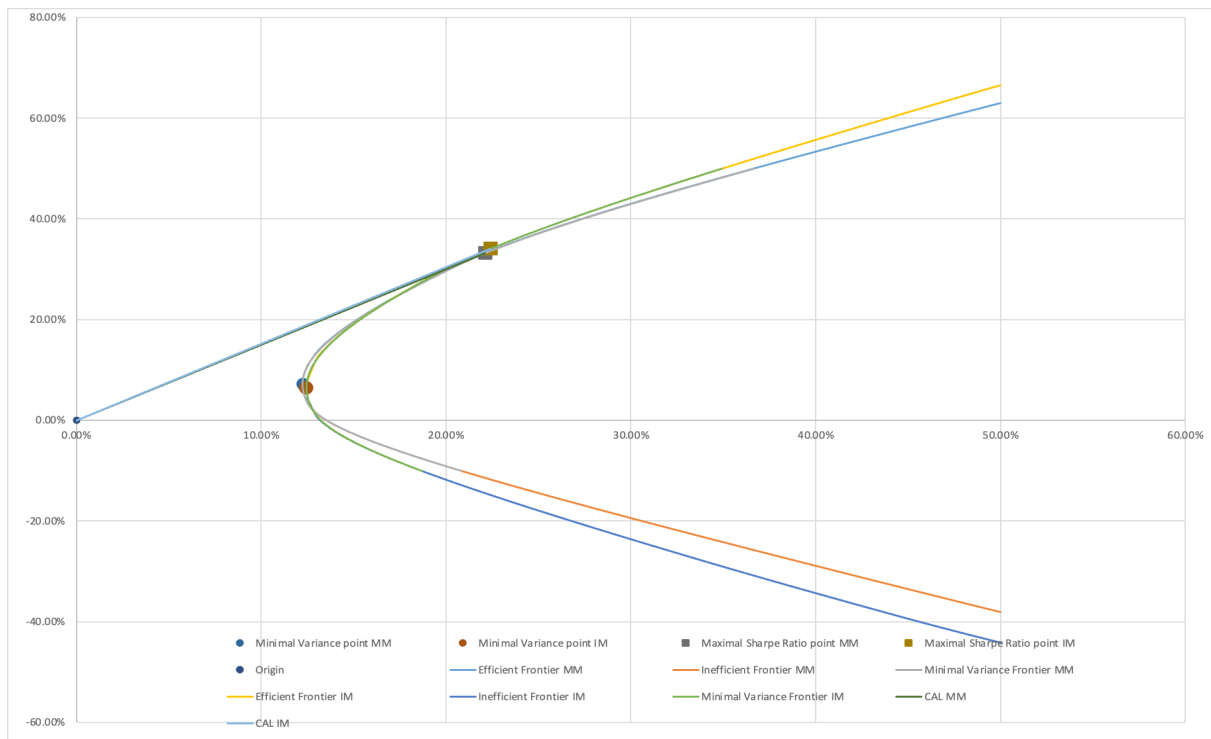
### 7.1 Comparison between the full Markowitz Model and the Index Model

We will use Solver Table to draw five pictures containing efficient frontier, inefficient frontier, minimal variance

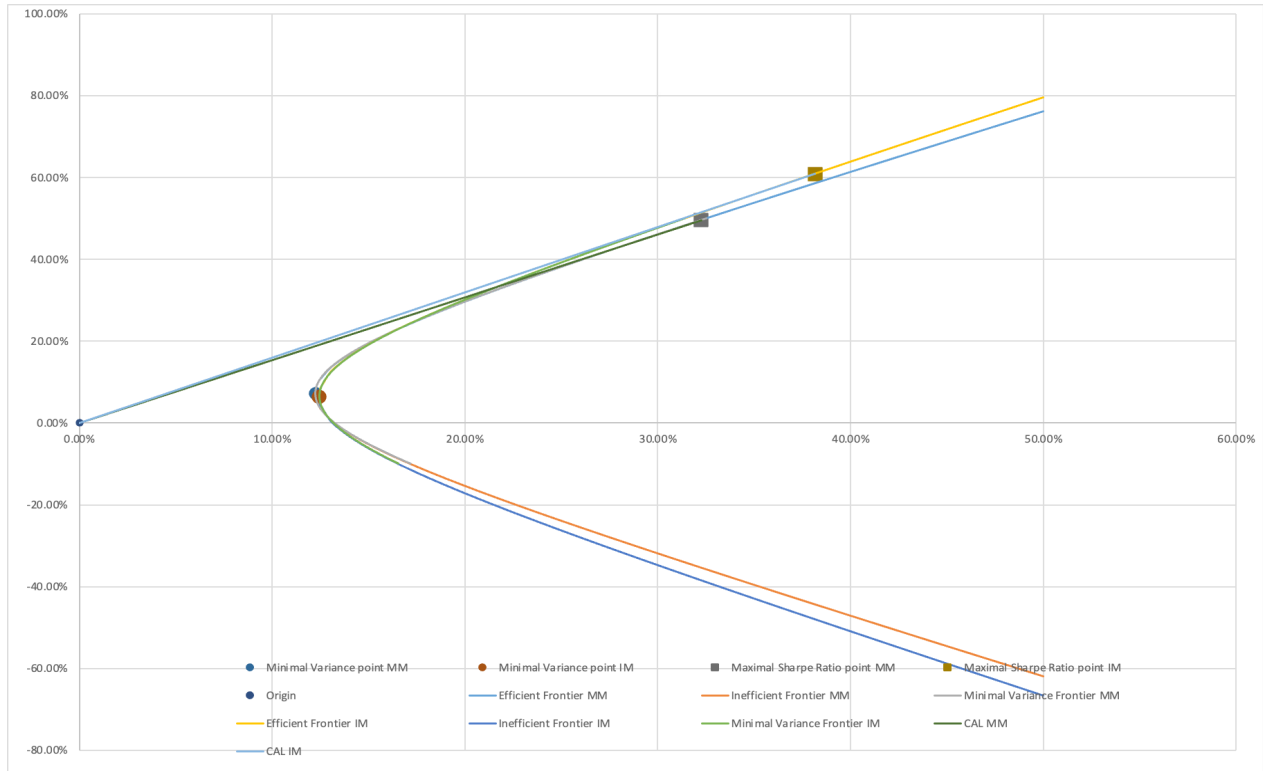
frontier, minimal variance points, and capital allocation based on five additional constraints.  
lines of the full Markowitz Model and the Index Model



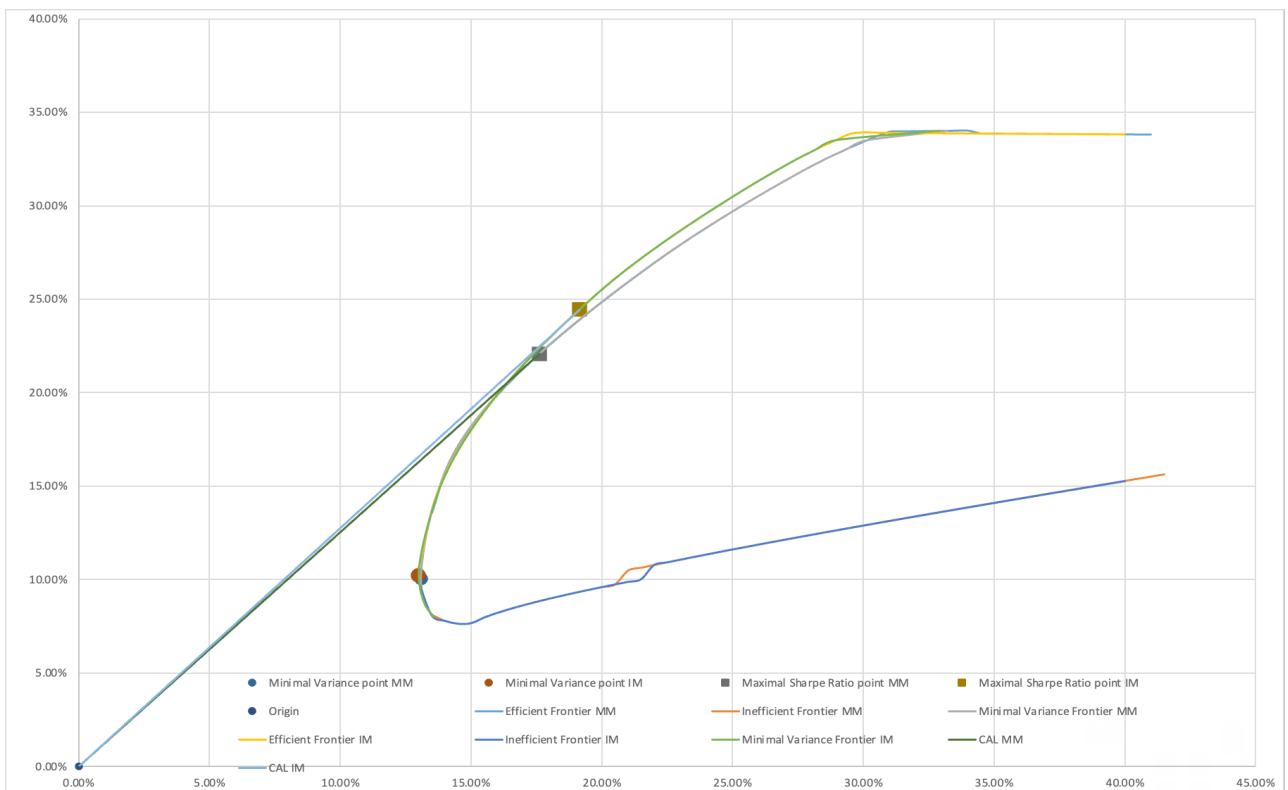
**Figure 11 under constraint 1**



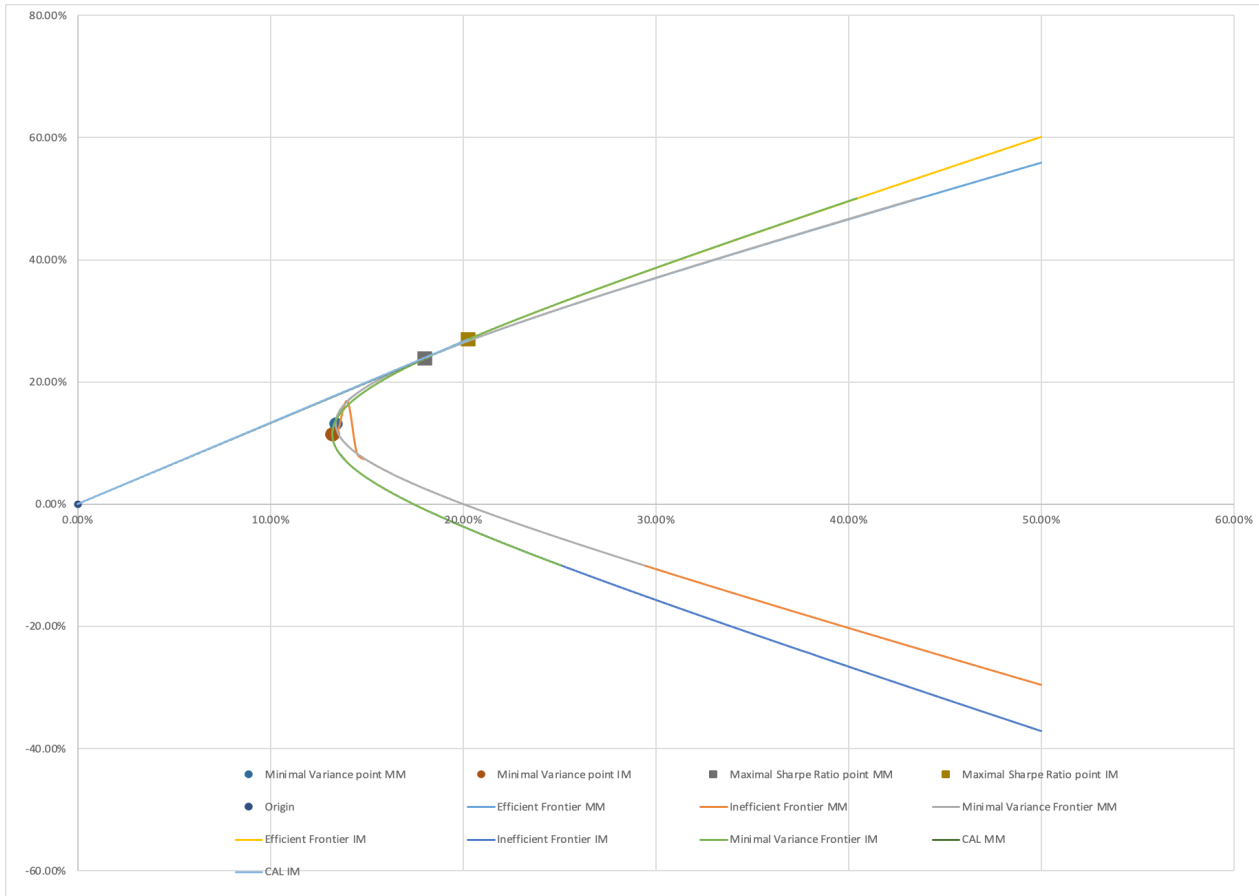
**Figure 12 under constraint 2**



**Figure 13 under constraint 3**



**Figure 14 under constraint 4**



**Figure 15 under constraint 5**

As seen in the five figures above, the efficient frontier of the two models under different additional constraints is similar. And, for the inefficient border under constraint 1, the maximal return of the full Markowitz Model is significantly higher than that of the Index Model for the same standard deviation. However, under constraint 4, the inefficient frontiers of the two almost completely overlap. Meanwhile, the Index Model calculates a greater risk and return for the optimal portfolio than the full Markowitz Model. This is probably because the Index Model ignores the correlation between stock returns.

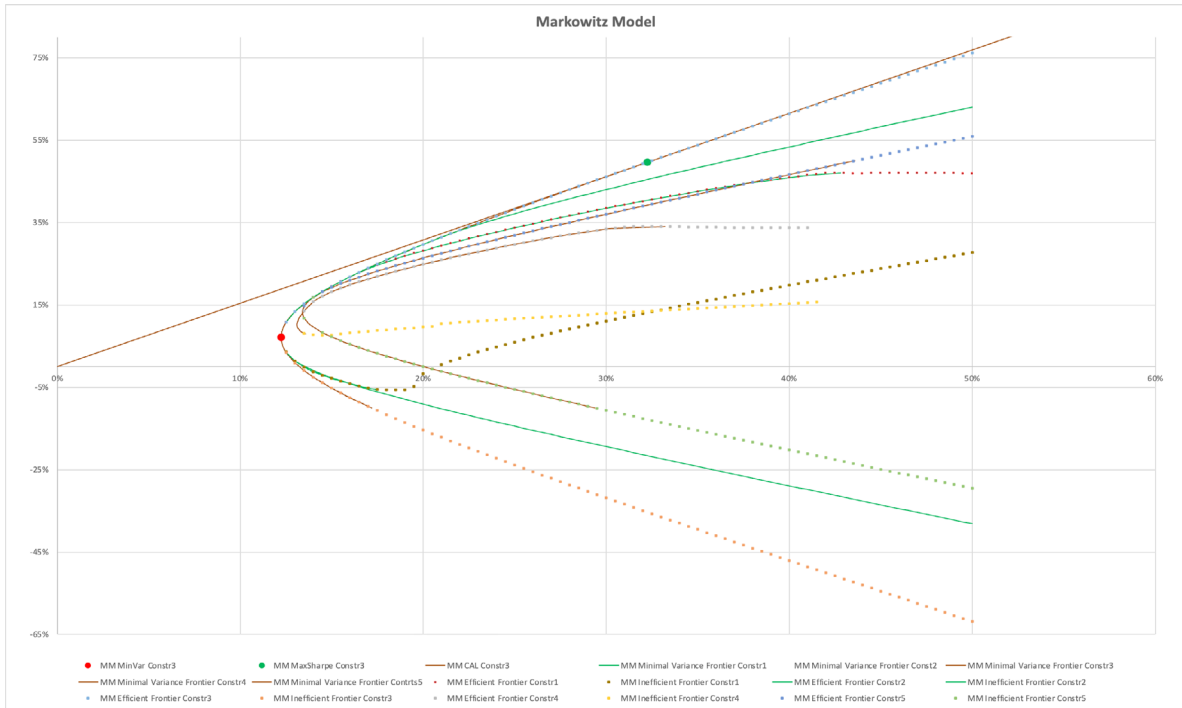
## 7.2 Comparison of five additional constraints

Here we will use Solver Table to draw two pictures containing the efficient, inefficient, and minimal variance frontier under each additional constraint based on the full Markowitz and the Index Models. Since we have learned above that compared to the other additional constraints, the standard deviation is the smallest, and the Sharpe

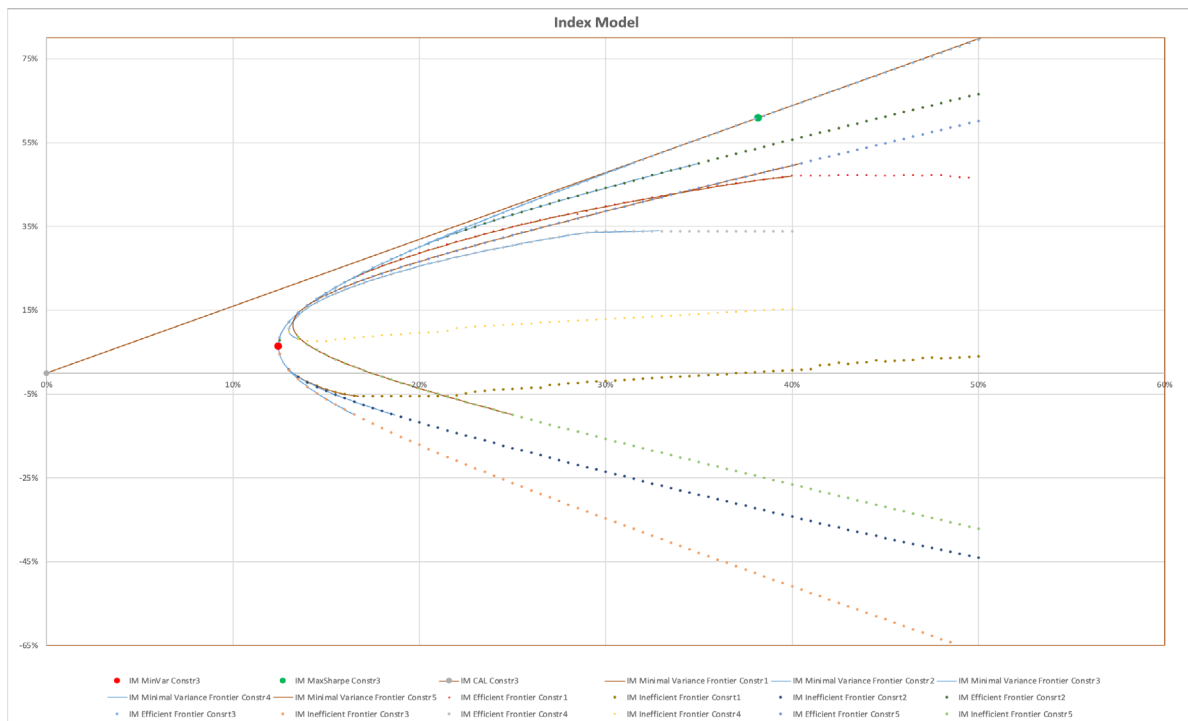
Ratio is the largest under constraint 3. Therefore, we will add a minimal variance point and capital allocation line of two models under constraint 3 to these two pictures.

As shown in the two figures above, the inefficient frontier under constraint one and constraint four slopes toward the upper right corner, which is different from the other cases. The inefficient frontier under constraint 2, constraint 3, and constraint 5 shows almost a symmetric shape with the efficient frontier.

And both the full Markowitz Model and the Index Models have an efficient frontier above other efficient frontiers and an inefficient frontier below other weak frontiers under constraint three compared to other additional constraints. This illustrates that additional constraints make the efficient frontier and inefficient frontier close to each other. That is, for the same risk, the additional constraint increases the minimal return of the portfolio. Still, it decreases the maximal return of the portfolio, resulting in a more concentrated risk-return distribution of the portfolio.



**Figure 16 under the full Markowitz Model**



**Figure 17 under the Index Model**

## 8 Conclusion

Asset allocation is an investment strategy generally based on an investor's risk appetite. It is designed to improve the return-risk ratio of a portfolio by defining and selecting various asset classes and evaluating their historical and

future performance to determine the weight of each asset class in the portfolio. Diversification of asset classes and specific investments is the core of asset allocation. The main objective is to seek a better return-risk profile based on the investor's own risk appetite and expectations and

achieve higher returns and lower risk over a longer time horizon. This article aims to study the application of the full Markowitz Model and the Index Model in the real stock market. We can calculate the relationship between portfolio return and risk by modeling some portfolio indicators. Investors can find their optimal portfolio from our calculations based on their investment preferences.

First, we collected data for ten stock indices from different sectors from 2001 to 2021 and calculated their daily returns. After plotting the Quantile-Quantile figure, we found that the daily returns did not obey a normal distribution, so the data had to be processed. Based on the central limit theorem, we transformed the daily returns into monthly returns. We plotted the Quantile-Quantile figures and found that the monthly returns could obey a normal distribution. Then, we calculated the annual average return, annual standard deviation, beta, alpha, and residual standard deviation for the ten stocks based on the monthly returns. We also used Python to plot a matrix heatmap of the correlation coefficients among the ten stocks. Then, based on the real stock market with portfolio investment, we introduced five additional constraints and transformed them into mathematical formulas based on stock weights. Finally, we used Solver and Solver Table in Excel to compute linear programming for portfolio indicators based on different models with different additional constraints. We presented the results visually, plotting the efficient frontier, inefficient frontier, minimal variance frontier, minimal variance points, and capital allocation lines and compared the results. We find that the minimum variance point of the portfolio is the smallest, and the Sharpe Ratio is the largest under constraint three compared to the other additional constraints. And for the same standard deviation, the efficient frontier under constraint 3 is above the other efficient boundaries, and the inefficient frontier is below the other inefficient frontiers.

Meanwhile, the inefficient borders under Constraint 1 and Constraint 4 show a tendency to slope upward to the right. Moreover, for any of the additional constraints, the maximal Sharpe Ratio calculated by the Index Model is larger than the Sharpe Ratio calculated by the full

Markowitz Model. Still, the risk and return corresponding to the optimal investment point are also more significant. However, some things could be improved in this article. The first is the deficiency in sample selection. This paper selects ten stocks for the portfolio analysis, which is too small a sample size compared to the total number of stocks in the stock market. Also, the stocks cover a limited number of industries. Moreover, these stocks are from companies with high visibility. This means that we cannot select a sample with a large enough sample size and coverage, which might lead to bias in the estimation results. The second is the deficiency of the model itself. Neither the complete Markowitz nor Index Model considers investors' investment preferences. At the same time, both models estimate the parameters based on historical data, which may be biased. Moreover, the use of standard deviation to measure risk has some limitations. Finally, the rational man assumption and the complete market assumption of both models are often difficult to be realized in real life.

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