ISSN 2959-6130

Compare the Black-Scholes Model, Monte Carlo Model, One Step Binomial Tree Model through the European options of Apple

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Abstract:

In this article, there are three Models, including Black-Scholes Model, Monte Carlo Model and the One-Step Model for Binomial Tree used to estimate the option premium of Apple, finding the most fitted model for estimating the European call and put options. Using the historical data of Apple in the recent year, the data of three models are found in Yahoo Finance. Simulating the possible outcomes in the future for 1000 steps and calculating the integral for the partial differential equation, spreadsheet is used for calculating the European call and put option premium by using the three models. Comparing the outcomes of three models, Black-Scholes Model provide a more accurate estimations on the European Options compared with the other two. The result indicates the pros and cons of three models, explaining specifically for the Monte Carlo Model, Black-Scholes Model, and One Step Binomial Model in both perspective of finance and mathematics.

Keywords: Monte Carlo Model; Black-Scholes Model; One-step Binomial Tree Model; European Option premium; Apple.

1. Introduction

1.1 Research Background and Significance

Increasing the positive incentives and loyalty of the employees, options is a kind of incentive mechanism to motivate employees to contribute more to the company. Looking forward to receiving a good rate of returns, employees are more focused on the long-term growth of the company. The European call options have been released by the company as a reward to the employees, retaining the top talent and encouraging them to stay with to company longer. employees are more luckily to work in a company for a long term on the grounds that the European options have the time to maturity for one month or even longer. Compared to using stocks or even cash, the options have better flexibility and efficiency. The cost of releasing the options is much lower than the cash and stocks, which means that influence or financial situations are also tiny. Even though the European call-and-put options are traded at a lower price, the amount of the option released is still restricted. Releasing the European call and put options, the company adjusted their stock in an interval based on the strike price they designed, making, dear volatility of stocks decreases. When the stock price is stable, the supply of employees' salaries is also stable and their financing risks will be decreased. Option pricing are not only use for the stocks, having the type of real options that are more likely to help people to do the financial decision making. Including the timing options, abandonment options, and expansion options, options are more likely to evaluate the adjusted present value of the project. It is obvious for investors to decide the time to start a project or whether have a capital expenditure. Using the Black-Scholes Model, Monte Carlo Model, and One -step Binomial Tree Model to analyze the option premium, this article compared the result difference between three models in a certain case.

1.2 Literature Review

Options, generally is a right that help investors to hedge their risks for their portfolios or even the company. Holger Kraft analyze the relationships between firm value and a firm's growth options, concluding that there is a positive relationships between the Tobin's Q increases and firm-level volatility [1]. There are 3 famous models which are Black-Scholes Model, Monte Carlo Model and Binomial Tree Model are generated based on the Mathematics. Daniel de Souza Santos and Tiago Alessandro Espinola Ferreira states that Black-Scholes Model is a parabolic deferential equation with the initial boundary, analyzing the network can solve the Black-Scholes Model based on the real-world stock options time series [2].

Based on the constant volatility of calculating the option premium, Aimmatul Ummah Alfajriyah et al. analyze the relationships between Black-Scholes Model and Binomial Tree Model. They claim that the implied Binomial Tree Model which is based on the Balck-Scholess Model indicates a smaller uncertainties compared to the Binomial Tree Model [3]. Constructing that volatility surfaces being essentially time-invariant, Ryan McCrickerd and Mikko S. Pakkanen claim that Monte Carlo Model is absence of analytical European Option Premium [4]. However, Victor Platon and Andreea Constantinescu states that Monte Carlo simulation method contains all the possible outcomes which generating several time under the normal distribution. Monte Carlo simulation method can be applied because of the widely recognized both by practitioners and the academic community [5]. Ting He et al. Comment that the basic model of Binomial Tree Model is created by Cox, Ross, and Rubinstein in 1979. Assuming that price of the underlying asset follows the binomial distribution with a constant probability, Binomial tree model contains a factor called risk-neutral probability [6]. Based on the theory claimed in 1979, Yossi Shvimer and Avi Herbon use different types of binomial models to estimate the levels of tradability, proximity to market (RMS) prices and profitability, concluding that moneyness is directly proportional to models' prices to actual market prices [7]. Except the three models states based on mathematics, there are several connections are built with different subject. Connecting the subject to Machine Learning, Shuaiqiang Liu et al. proposes a data-driven approach through Artificial Neural Network (ANN) to evaluate the implied volatility [8]. Similarly, Gilles Pagès et al. demonstrate the generic framework to compute any greeks and apply it on different types of financial contracts, including multidimensional Basket Call, European options, American options and stochastic volatility models such as Heston's model [9]. In the physics perspective, Patrick Rebentrost et al. indicate the method of applying the amplitude estimation algorithm to speed up the quadratic quantum, providing a starting point for further research at the quantum computing and finance [10].

1.3 Research Contents

In this article, the result calculated based on the Monte Carlo Model, Black-Scholes Model, and Binomial Model, comparing and analyzing the result between the Models and providing an advice for different investors.

2. The European Call and Put Options Trading by Apple Stock

Based on the sustainable development of the technology industry, occupied a big part of the market share. Being an advanced technology company, Apple is good at creating mobile phones, iPads, laptops etc. Apple has a sort of key patent that can help them to increase the returns for keeping innovating. Considering the perspective of Apple, it is more sustainable for them to have both equity and debt in their capital structure. Hedging the risks of the companies, the options can prevent their big loss in particular situations. It will be analyzed in both aspects of internal and external of the company.

The price of European call and put options will be priced based on the elements. Including the time to maturity, volatility, dividend shields etc. Based on the daily historical data indicated on Yahoo Finance, there are 252 daily returns and a daily standard deviation. The Table1 illustrates the returns and standard deviations daily and annually. The risk-free rate calculated is based on the return on 5 years of US Treasury Bills. The annual return for Apple is 3.33% and the Risk-free rate is 1.4%, which implies that the annual excess return is 1.93%. Apple enable a positive image through demonstrating their returns and excess return in positive rate.

Table 1. Standard Deviation, returns and risk free rate

Daily standard Deviation	Daily Returns	Daily Risk Free Rate
12.40%	0.009%	0.12%
Annual Standard Deviation	Annual Returns	Annual Risk Free Rate
19.69%	2.32%	1.40%

By distributing the dividend of 0.24, Apple enables a quarterly dividend shield quarterly based on the Table 2. When there is a dividend distribution in Apple, the stock price of that particular date will decrease in that particular date. If Apple regularly distributes dividends, this may affect the incentive effect of options, especially for the European call options. Expressing a positive signal of Apple's operating, the stock price will increase and investors are more likely to buy the European call options rather the the European put options. The price of the options will be influenced based on the Non-Arbitrage Principle, having the same value as the Option Payoff on the maturity date.

Diminishing, especially for call options, the incentive effect of dividend payouts on the grounds that the action reduces the potential for stock price appreciation, therefore the intrinsic value of the options also decreased.

Table 2. Dividend payout in quarter

2024/2/9	0.24 Dividend
2023/11/10	0.24 Dividend
2023/8/11	0.24 Dividend
2023/5/12	0.24 Dividend

In this particular part, the price of the European call options and European put options will be indicated in three given models, including the Black-Scholes model, binomial tree model, and Monte Carlo mode in the perspective of a company and individual investors.

2.1 Monte Carlo Model

To estimate and predict the option premium precisely, the strike price is set as $165.00 which indicated in Table 3. Analyzing both the call and put position, the initial stock price will be equal to the last historical daily data on 1st June 2024. There are a total of 1000 simulations and 100 random walk for each simulation. Stimulating the possible future situations by creating large amounts of random samples, the Monte Carlo Model creates random paths for each stimulation accurately. Geometric Brownian motion is more likely to diversify the risks, providing a significant result.

Table 3. Data use in Monte Carlo Model

Price of stock on 1st June	169.3
Risk (volatility)	19.69%
Risk free rate	1.40%
Dividend (Quarterly)	0.24
Time to maturity	1
Option strike price	165
Number of stimulations	1000
Time increment	0.0039
Number of steps	100

2.2 Black-Scholes Model

There are several assumptions made, including an efficient market, no dividend payment, constant volatility, and continued compounding interest. The d1 and d2 are calculated based on the other data given on Table 4 through using the standard normal cumulative distribution function. Taking the integral form negative infinity to d1 or d2, European call and put options can be calculated based on the formula.

Table 4. Data used in Black- Scholes Model

Price of stock on 1st June	169.3
Risk (volatility)	19.69%
Risk free rate	1.40%
Time to maturity	1
Option strike price	165
d1	0.3004
d2	0.1035

2.3 Binomial Tree Model

The one step Binomial Tree Model was used for the approximation of option premium, including two possibilities of going up or down under binomial distributions. The up and down factors are used to calculate specifically for the payoff both call and put options, indicating can calculated based on the data labeled in Table 5. Based on the law of one price, the current price of the option premium equals the present value of the future cash flow of the option payoff.

Table 5. Data used in 1-Step Binomial Tree Model

Price of stock on 1st June	169.3
Risk (volatility)	19.69%
Risk free rate	1.40%
Time to maturity	1
Option strike price	165
Time increment	0.0039
u	1.0124
d	0.9876

3. The Result Comparison Between Three Model

Enabling a big difference in pricing in Table 6, one step Binomial Tree Model indicates a big difference compared with the other two models on the grounds that it only provides two possible outcomes either going up or down. The option payoff only depends on the risk natural probabilities, indicating a limitation of including simulations for unknown future situations. The outcomes of the Monte Carlo Model and Black-Scholes Model are relatively close, using the Wiener process for approximating the option premium. The difference caused approximately $4 in the call option and $5 in the put option because of different assumptions. Assuming the stock price is under the normal distribution, the Monte Carlo Model includes the dividend payment which the Black-Scholes Model does not. Black-Scholes contains the assumptions of an efficient market etc., having less flexibility to adapt to different situations.

Table 6. Result comparison of three models

	Monte Carlo model	Black-Scholes	Binomial Tree
Call Option premium	20.04377645	16.58480176	8.527988053
Put Option premium	14.76098719	9.98431874	0

From the point of view of individual investors, they are more likely to invest in the option of the stock that demonstrates high volatility, having a higher possibility of receiving a positive return rate by using a smaller amount of money. The minimum option payoff for the European call and put options is $0, losing the option premium at the maximum. Distributing the dividend payments in time, Apple has a stable returns in recent years. The positive signal released derived from the dividend payment of Apple, promoting the European call options to become more popular for individual investors. It is suitable for a risk-averse investor to add European Options into their portfolios to hedge the risks, which implies that the portfolios will be diversified and reduce the risks.

In order to hedge the risks of investing the Apple's stock, it is more suitable for investors to create a portfolio which contain not only the European options and stocks but also include the bonds or other stocks with the negative correlations to Apple's stocks. There are two kinds of risks, including the systematic risks and unsystematic risks. Especially for the risk adverse, investor have a preference of investing the underlying assets and portfolios with the highest returns at a given risk level, through finding the maximum Sharpe Ratio.

4. Conclusion

Comparing the output of three models and the reference above, three models which are Mote Carlo Model, Black-Scholes Model and One-Step Binomial Tree Model indicates a approximation for the European call and put

options based on the same period historical data of Apple. Black-Scholes Model performed the best in estimating the European option premium. Binomial Tree Model is the one that provide outlier outcomes with less accuracy when the same conditions provided. However, Monte Carlo Simulation ignore the risks of time when it randomly stimulate the possible future risks, which is not suitable for the European options because the European options only available on the maturity date. Black-Scholes Model, simultaneously, provide a better approximation through considering different situations under the standard normal distribution.

In this article, there is one-step Binomial Model be analyzed through the data which is a reason that causes the wired data. The Apple's historical data is only analyzed for the recent one year, indicating in the Yahoo Finance Website. Providing a good choice for estimating the European option premium, this article analyzing and building up the connections between mathematics and finance though using the partial differential equations etc. Illustrating the difference between Models, the article helpful for investors or scholars to find the merits and demerits for the models in the future.

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